

**The Development of Rotating, Non-linear
Numerical Models (GF2, GF3) Simulating
Barotropic Mixed Tides in a Complex Coastal
System Located Between Vancouver Island
and the Mainland**

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ABSTRACT

Crean, P.B. 1983. The Development of Rotating, Non-Linear Numerical Models (GF2, GF3) Simulating Barotropic Mixed Tides in a Complex Coastal System Located Between Vancouver Island and the Mainland. Can. Tech. Rep. Hydrogr. Ocean Sci. 31: 65 p.

The systematic investigation of dynamical assumptions, locations of open boundaries, frictional dissipation and schematic representation of topography led to the development of a non-linear rotating numerical model, comprising joint one- and two-dimensional schemes, which could satisfactorily simulate barotropic mixed tides in a highly complex coastal system. This system consists of a deep inner strait (Strait of Georgia) which is connected to the open ocean at its northern end through a constricted system of passages and tidal rapids, and at its southern end around a 180° bend containing a further system of channels and passes which lead to an outer strait (Juan de Fuca Strait). An extensive complex of side channels (Puget Sound system) opens off the outside of this bend. The degenerate amphidromic systems of the semi-diurnal tidal constituents are located in the inner part of this outer strait. Minimal dependence of the computed tides on elevations prescribed from observations at boundary openings was required before a satisfactory distribution of frictional dissipation could be achieved. It was then shown that this frictional dissipation induced a significant interaction between the dominant M_2 and K_1 tidal constituents.

Illustrative of the performance of the model (GF2), comparisons are presented between observed and computed mixed tides and streams over a 30-day period for the representative locations of this model.

Elevations obtained from this model were then employed to provide boundary conditions for a more detailed (2 km mesh) model (GF3) of the strongly tidal southern part of the region.

key words: Tides, numerical model, Strait of Georgia.

RÉSUMÉ

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L'étude systématique d'hypothèses dynamiques, des emplacements des limites, de la dissipation par frottement et de la représentation schématique de la topographie ont conduit à l'élaboration d'un modèle numérique rotatif non linéaire, comprenant des schémas reliés à une et deux dimensions, qui pourrait simuler de façon satisfaisante des marées mixtes barotropiques dans un système côtier très complexe. Ce système consiste en un détroit intérieur profond (détroit de Géorgie) qui est relié à la mer libre à son extrémité nord par un système plus étroit de passages et de rapides tidaux, et à son extrémité sud par une courbe de 180° contenant un autre système de chenaux et de passes qui mènent à un détroit extérieur (détroit de Juan de Fuca). Un ensemble considérable de chenaux latéraux (système de la baie Puget) s'ouvre au large à l'extérieur de cette courbe. Les systèmes amphidromiques dégénérés des éléments tidaux semi-diurnes sont situés dans la partie intérieure de ce détroit extérieur. Pour obtenir une distribution satisfaisante de la dissipation par frottement, on a dû effectuer certains calculs des marées à partir des hauteurs tirées d'observations aux ouvertures limites. Il a été alors démontré que cette dissipation par frottement a produit une interaction importante entre les éléments tidaux dominants M_2 et K_1 .

Pour illustrer le rendement du modèle (GR2), on présente des comparaisons entre les marées mixtes et les courants observés et calculés sur une période de 30 jours pour les emplacements représentatifs de ce modèle.

Les hauteurs obtenues à partir de ce modèle sont alors utilisées pour fournir des conditions limites pour un modèle (GR3) plus détaillé (réseau de 2 km) de la partie sud de la région fortement soumise à l'influence de la marée.

Mots-clés: marées, modèle numérique, détroit de Géorgie.

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1. INTRODUCTION

In recent years considerable success has been achieved in the use of hydrodynamical numerical methods to simulate barotropic tides in coastal seas and estuaries. Though the basic procedures are relatively simple, difficulties are encountered in the application of such techniques to problems concerning strong tidal flows in topographically complex systems. It is usually presumed that variations in phase speed and partial reflections of a long wave propagating through the system can be adequately represented by a sufficiently detailed finite difference approximation to the actual geometry. Energy losses from the wave are then accounted for empirically by adjusting the frictional coefficients in the momentum equations to optimize the agreement between the observed and computed tidal elevations around the interior of the sea. Before conclusions are drawn concerning the dynamics of wave propagation it is important to ensure as far as possible that such adjustments do not compensate for inadequacies in the theoretical assumptions, schematic representation of topography, or in prescribing boundary conditions. A prerequisite is thus a sufficiency of observations, both elevations and velocities, to establish standards of performance which could in principle be achieved by the model. If necessary, the sensitivity of the computed fields to the various approximations is then systematically investigated, and improvements made, until these standards are attained or the need for a radically different type of model has been demonstrated.

This paper describes the development of a model simulating the propagation of mixed tides in a highly complex coastal system located between Vancouver Island and the mainland coasts of British Columbia and the State of Washington. The reasons for the study devolve essentially on management problems in connection with the important commercial and sports fisheries of the region, the disposal of domestic and industrial effluent together with various more local problems in coastal engineering.

The system is estuarine in character (Waldichuk 1957, Herlinveaux and Giovando, 1969) and the eventual aim of this work is a full three-dimensional simulation of the main features of the estuarine circulation and its response to forcing by winds, changes in fresh-water inflow or in conditions on the adjacent continental shelf. The distribution of salinity is, however, markedly dependent on the regionally varying mixing by the tidal streams. Thus the baroclinic circulation could be contingent, not only on residual flows deriving from non-linear aspects of the surface tides, but also on the mixing processes occasioned by their associated streams.

The basic geographical features pertinent to a discussion of tides in the region are now described using approximate general dimensions. There is an outer strait (Juan de Fuca Strait), Figure 1, 145 km in length, 22-30 km wide, and in which depths decrease gradually from some 300 m near the entrance to about 100 m in the channel between the shoals in its inner end. The inner strait (Strait of Georgia) is 220 km long, of average width 33 km and with maximal depths along the channel of order 150-400 m. Between the inner and outer straits there is a complex system of channels and passes between the San Juan and Gulf Islands, Figure 2. Two main conveying channels are distinguished. The largest of these include Haro Strait and Boundary Passage. Haro Strait is 50 km long and some 10 km wide. The western part of the strait is relatively shallow but along its eastern side there is a

channel with depths of order 180-250 m and width about 3 km. This deep channel turns sharply eastward to Boundary Pass which connects with the inner strait. The second principal conveying channel is Rosario Strait, 30 km long, about 4-10 km wide and with depths of order 50 m.

The northern end of the inner strait is joined to the open ocean by a system of narrow passages which merge into a single strait (Johnstone Strait) between Vancouver Island and the mainland. The largest single conveying channel follows the Vancouver Island coastline and includes Johnstone Strait and Discovery Passage. The part of this channel which features in the model is 100 km long, 2-3 km in width and some 200 m deep except in the vicinity of a sill of depth about 50-100 m. All of these northern passages are characterized by relatively shallow sills.

In addition to these channels there exist a number of mainland inlets. The largest and most important of these is the Puget Sound system with its primary entrance located just seaward of the island group between the inner and outer straits. A much smaller secondary entrance, Deception Pass (Fig. 1), also exists but this was shown by model trials to be of only local significance. This region has a surface area about one-third that of the Strait of Georgia, and an overall length of some 175 km. Depths in Puget Sound itself are generally in excess of 200 m, though there exists a sill of about 40 m at the entrance.

The remaining smaller inlets open off the mainland coast of the Strait of Georgia and in some instances are distinguished by depths in excess of 650 m, an important factor in the selection of the time step for explicit models.

Tides in the system vary geographically from mixed mainly diurnal (Victoria, Fig. 1) to mixed mainly semi-diurnal (Vancouver). The basic features of the major semi-diurnal (M_2) and diurnal (K_1) tides may be illustrated by the co-amplitude and cophase diagrams shown in Figure 3. These diagrams are based on computed elevations from the model.

The essential features of the tidal problem are now briefly reviewed. Comparisons of the observed slopes of the water surface and those computed for the equilibrium tides indicate the relative unimportance of direct gravitational forcing compared to that of the ocean tide at the open boundaries of the system. To a first approximation, the resulting tidal constituent co-oscillations consist essentially of the superposed incoming Kelvin waves through Juan de Fuca Strait and the reflected waves from the northern end of the Strait of Georgia. The actual distributions of harmonic constants will however be affected by energy dissipation, asymmetry of channel geometry, partial reflections from the islands between the inner and outer Straits, reflective conditions in northern Georgia Strait (Hendershott and Speranza, 1977), and the generation of co-oscillations in the various mainland inlets. Also, there may exist significant non-linear effects associated with advective accelerations and curvature of the flow field between the inner and outer straits or with friction. More particularly, incorrect simulation of the wave propagation and reflection through the outer and inner straits will bring about incorrect distributions of range and phase in the important semi-diurnal amphidromic systems; thus adversely affecting the tidal regime in the Puget Sound system.

The only previous theoretical work on barotropic tidal propagation between Vancouver island and the mainland was undertaken by Redfield, 1950. It was assumed that the basic features of the motion could be represented by the superposition of damped incoming and reflected progressive waves in Juan de Fuca Strait and the Strait of Georgia, the northern end of the latter being assumed closed. In applying wave relations deduced for uniform channels to a real system it was assumed that the effect of irregularities in the cross-section and wave velocity is to alter the geographical distribution of phase differences, that damping is proportional to phase change along the channel, that the damping is linearly related to velocity and that the damping coefficient is constant along the channel. It was concluded that the general features of both the main diurnal and semi-diurnal constituents were consistently correlated by the theoretical treatment and that the degree of damping required to optimize the agreement between observed and calculated tidal constants of both species was greater than that encountered in applications of the same technique to the Bay of Fundy and Long Island Sound.

The present work commenced with some preliminary calculations on the tidal currents likely to be encountered in the Strait of Georgia. Numerical evaluation of the M_2 tidal current ellipses following Taylor (1921), due to Kelvin wave reflections which occur in a rotating rectangular gulf, of depth and width roughly that of the Strait of Georgia, indicated that currents of the rectilinear type should dominate except in a highly limited area near the closed end. Using known surface areas and cross-sections and assuming a single M_2 tidal amplitude at a closed northern end, the average rectilinear tidal currents over each cross-section could readily be determined along the Strait of Georgia (Proudman, 1953). Any attempt to extend these calculations beyond the San Juan Islands led to large discrepancies between the observed and calculated amplitudes.

At this time, access to a small computer permitted a preliminary program of simple numerical trials. The main conveying channels and mainland inlets of the overall system were represented by a scheme of joined one-dimensional channels. The Northern Passages (Fig. 1) were represented by a single channel. Using central finite difference formulations of the continuity and momentum equations (Rossiter and Lennon, 1965), the elevations and average velocities over a cross-section were calculated (Crean, 1969).

Friction coefficients were adjusted to optimize the agreement between the observed and computed tidal constants for the main semi-diurnal (M_2) and diurnal (K_1) tides. The simulations of the M_2 and K_1 tides provided by this model were encouraging. The main features of both constituents along the major conveying channels were substantially correct, in particular the location of the semi-diurnal degenerate nodal region in the inner part of Juan de Fuca Strait. With a frictional dissipation based on optimal simulation of the M_2 constituent amplitudes and phases the following shortcomings were noted. The computed M_2 tidal phase lags in the Strait of Georgia were too small by about 20° - 30° and the amplitudes in the Puget Sound system low by about 50%, though in this latter instance the phase lags were correct. The computed K_1 tidal amplitudes in the Strait of Georgia were low by about 10%, the phase lags too small by about 10° , while the amplitude in Puget Sound was low by about 10%, though the phase was correct. The friction coefficient giving these results was roughly ten times that employed in numerical work on other coastal seas. This was qualitatively consistent with

Redfield's findings, but still appeared suspiciously large. The relation between computed elevations and velocities in the mainland inlets conformed essentially to standing wave behaviour.

A point of interest concerns the wave reflection problem at the northern end of the Strait of Georgia. If this model is operated with the northern end of the Strait of Georgia closed, but with Bute Inlet (Fig. 1) included in the model, an appreciable shift of the M_2 nodal region towards the head of the system occurs. If no change in the tidal regime occurred the irrelevance of the flows through Northern Passages could be inferred. Little else, however, can be deduced from this trial since the boundary conditions at the entrance to Juan de Fuca Strait may not be applicable.

It was clear from these numerical experiments that no adjustment of friction could improve the model performance and that a two-dimensional model, which could include geostrophic effects in the inner part of Juan de Fuca Strait offered considerable promise of reliably simulating tidal dynamics within the system. This paper concerns the development of such a model.

2. OBSERVATIONS

In collaboration with the Canadian Hydrographic Service (Pacific Region), measurements of tides and streams were undertaken for purposes of operating, adjusting and verifying the model. Additional data were obtained from the National Oceanographic and Atmospheric Administration, U.S. Bureau of Commerce. The locations of tide gauges and current meter arrays are shown in Figure 4.

Simultaneous records were obtained over a period of one year from the gauges installed to supplement the permanent stations (Harmonic Constants and Associated Data for Canadian Tidal Waters, Vol. V, 1973). Extensive records, generally of 29 days length, are available for numerous other locations throughout the system. Harmonic constants obtained from these latter are less consistent than those obtained from one-year records and their use has been limited to certain one-dimensional channels where no other data were available.

Each of the current meter arrays, shown by solid lines in Figure 4, generally involved several sub-surface moorings, each supporting a vertical sequence of some 2 to 4 meters. Record lengths obtained were usually about 29 days. The magnitudes of the tidal streams increase progressively from line 2 to line 6. Of particular interest are lines 1 and 11 which are located near the two open boundaries where elevations are prescribed to operate the model. These lines thus provide a good check on the performance of the model. With the exception of line 14, more detailed information, including harmonic analyses, are available (Data records of current observations, 1967-1975).

Velocity records exceeding one year in length have also been obtained for a cross-section of the Strait of Georgia (Stations H-06, H-16, H-26, Fig. 4), and for a location in Juan de Fuca Strait in mid-channel south of Race Rocks (Station 001). (The latter record includes two 8-month sequences

separated by a 6-week gap.) Detailed spectral analyses of these Strait of Georgia data have been reported (Chang and Pond, 1976).

3. FUNDAMENTAL HYDRODYNAMICAL EQUATIONS

Three types of numerical scheme are employed. The Straits of Georgia, Juan de Fuca Strait and the intervening system of islands and channels are represented by a two-dimensional, vertically integrated numerical scheme. To represent narrower channels where geostrophic effects are not important the first one-dimensional scheme, which is applied to the Puget Sound system, is an adaptation of the two-dimensional scheme referred to above. The second one-dimensional scheme is considerably more versatile and better adapted to particular features encountered in the Northern Passages and other mainland inlets.

(a) Two-dimensional scheme

The vertically integrated equations of continuity and momentum applicable to tidal motions in a flat rotating sea may be written (Proudman, 1953):

$$\frac{\partial \zeta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left[\frac{U^2}{\zeta+h} \right] + \frac{\partial}{\partial y} \left[\frac{UV}{\zeta+h} \right] - fV + g(\zeta+h) \frac{\partial}{\partial x} (\zeta-\bar{\zeta}) \\ + \frac{kU\sqrt{U^2+V^2}}{(\zeta+h)^2} = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial V}{\partial t} + \frac{\partial}{\partial x} \left[\frac{UV}{\zeta+h} \right] + \frac{\partial}{\partial y} \left[\frac{V^2}{\zeta+h} \right] + fU + g(\zeta+h) \frac{\partial}{\partial y} (\zeta-\bar{\zeta}) \\ + \frac{kV\sqrt{U^2+V^2}}{(\zeta+h)^2} = 0 \end{aligned} \quad (3)$$

where the notation is as follows:

x, y

Cartesian coordinates in the plane of the undisturbed surface

t

Time

$z(x,y,t)$	Elevation of the sea surface
$U(x,y,t)$ $V(x,y,t)$	Components of vertically integrated velocity in directions x and y
$h(x,y)$	Depth of water below the undisturbed surface
f	Coriolis parameter, assumed constant over the region
$\bar{z}(t)$	Equilibrium form of the surface elevation corresponding to the tide generating body forces, assumed constant over the region
$k(x,y)$	Coefficient of friction
g	Acceleration due to gravity

In the derivation of these equations it is assumed that the velocities are essentially independent of depth.

Equations (1), (2) and (3) are solved numerically for the sea area. The finite difference scheme is spatially similar to that used by Hansen (1961) but a modified numerical procedure in time is employed (Flather and Heaps, 1975).

The area is represented on a system of orthogonal grid lines. The grid notation is shown in Figure 5. Each mesh is of side 2 km and is characterized by a central elevation point while the normal components of vertically integrated velocity are evaluated at the mid-points of the sides.

To calculate the elevation $z_i(t+\Delta t)$ at the centre of a mesh equation (1) is replaced by:

$$(z_i(t+\Delta t) - z_i(t)) \frac{1}{\Delta t} = -(U_i(t) - U_{i-1}(t) + V_{i-n}(t) - V_i(t)) \frac{1}{\Delta x} \quad (4)$$

where n is the number of columns in the numerical grid scheme.

The momentum equations (2) and (3) are replaced by:

$$\begin{aligned}
& \left[U_i(t+\Delta t) - U_i(t) \right] \frac{1}{\Delta t} = f \bar{V}_i(t) \\
& - g \frac{\bar{H}_i^x}{\Delta z} \left[z_{i+1}(t+\Delta t) - z_i(t+\Delta t) \right] - K_i U_i(t) \frac{\left[U_i^2(t) + \bar{V}_i^2(t) \right]^{\frac{1}{2}}}{(\bar{H}_i^x)^2} \\
& - \left[\frac{\left[\bar{U}_i^x(t) \right]^2}{\frac{1}{2} [\bar{H}_i^x + \bar{H}_{i+1}^x]} - \frac{\left[\bar{U}_{i-1}^x(t) \right]^2}{\frac{1}{2} [\bar{H}_{i-1}^x + \bar{H}_i^x]} \right] \frac{1}{\Delta z} \\
& - \left[\frac{U_i^y(t) \bar{V}_{i-n}^x(t)}{\bar{H}_i} - \frac{U_{i+n}^y(t) \bar{V}_i^x(t)}{\bar{H}_{i-n}} \right] \frac{1}{\Delta z} + \quad (5)
\end{aligned}$$

$$\begin{aligned}
& \left[V_i(t+\Delta t) - V_i(t) \right] \frac{1}{\Delta t} = -f \bar{U}_i(t+\Delta t) - \frac{g \bar{H}_{i-n}^y}{\Delta z} \left[z_i(t+\Delta t) - z_{i-n}(t+\Delta t) \right] \\
& - K_i V_i(t) \frac{\left[\bar{U}_i^2(t) + V_i^2(t) \right]^{\frac{1}{2}}}{\left[\bar{H}_{i+n}^y \right]^2} \\
& - \left[\frac{U_{i+n}^y(t) \bar{V}_i^x(t)}{\bar{H}_{i+n}} - \frac{U_{i+n-1}^y(t) \bar{V}_{i-1}^x(t)}{\bar{H}_{i+n-1}} \right] \frac{1}{\Delta z} \\
& - \left[\frac{\left[\bar{V}_i^y(t) \right]^2}{\frac{1}{2} [\bar{H}_i^y + \bar{H}_{i+n}^y]} - \frac{\left[\bar{V}_{i+n}^y(t) \right]^2}{\frac{1}{2} [\bar{H}_{i+n}^y + \bar{H}_{i+2n}^y]} \right] \frac{1}{\Delta z} \quad (6)
\end{aligned}$$

where the averaging of a grid variable with the neighbouring variable of that type in the direction x or y increasing is denoted respectively by expressions such as:

$$\bar{U}_i^x = \frac{1}{2} [U_i + U_{i+1}] , \quad \bar{U}_i^y = \frac{1}{2} [U_i + U_{i-n}]$$

To denote the value at a point of a variable obtained by averaging the four values of that variable surrounding the point, the notation is:

$$\bar{H}_i = \frac{1}{4} (H_i + H_{i-n} + H_{i-n+1} + H_{i+1}) \quad \text{where } H_i = \zeta_i + h_i$$

$$\bar{U}_i = \frac{1}{4} (U_i + U_{i+n} + U_{i+n-1} + U_{i-1}) \quad \bar{V}_i = \frac{1}{4} (V_i + V_{i-n} + V_{i-n+1} + V_{i+1})$$

In laying out the model grid, coastal boundaries are represented on mesh lines when the normal component of velocity is zero. Open boundaries are taken along a line of elevation points where the values are determined from observations. Equations (4), (5) and (6) are solved explicitly at their respective locations in the interior of the grid from initial conditions $\zeta=U=V=0$. Under the influence of friction, the effect of the initial conditions becomes negligibly small after about two tidal cycles and the solution responds to the values prescribed at boundary openings. In earlier stages of the work the trials undertaken with this computational scheme proceeded from simpler forms of the equations (5) and (6), in which the advective accelerations and gravitational tide-producing forces were omitted. The influence of the latter was subsequently determined more carefully when it appeared that they might significantly reduce the relatively small differences between computed and observed tides.

(b) First one-dimensional scheme

The equations of continuity and momentum (Dronkers, 1964) as applied at a cross-section of a one-dimensional channel are:

$$\frac{\partial \zeta}{\partial t} + \frac{1}{b} \frac{\partial Q}{\partial s} = 0 \quad (7)$$

$$\frac{1}{A} \frac{\partial Q}{\partial t} + 2 \frac{Q}{A^2} \frac{\partial Q}{\partial s} - \frac{Q^2}{A^3} \frac{\partial A}{\partial s} + g \frac{\partial \zeta}{\partial s} + \frac{k Q |Q|}{A^2 (\zeta + h)} = 0 \quad (8)$$

where the additional notation is as follows:

s distance along the median line of the channel

$Q(s,t)$ volume flow rate through a cross-section

$A(s,t)$ area of a cross-section

- $h(s)$ average depth of a cross-section below the mean level
 $b(s)$ average storage width of the cross-section
 K coefficient of friction

Relatively small storage areas off the main channels are included in the average storage width. Changes in momentum flux deriving from temporal variations in cross-sectional areas and spatial gradients of the cross-sectional areas are included in equation (8).

In applying the first finite difference scheme to the Puget Sound system the assumptions that the advective accelerations were negligible and that the cross-sectional areas were independent of time led to satisfactory tidal simulations for present purposes. Thus, the second and third terms were omitted from (8).

The finite difference formulation of these equations conformed essentially to that used in the two-dimensional model. The computational grid is shown in Figure 6.

The finite difference forms of equations (7) and (8) are:

$$\left[\zeta'_i - \zeta \right] \frac{1}{\Delta t} = - \left[Q_i - Q_{i-1} \right] \frac{1}{b_i \Delta s} \quad (9)$$

$$\left[Q'_i - Q_i \right] \frac{1}{A_i \Delta t} = -g \left[\zeta'_{i+1} - \zeta'_i \right] \frac{1}{\Delta s} - \frac{K_i Q_i(t) |Q_i(t)|}{A_i^2 (\zeta_i + h_i)} \quad (10)$$

where:

- Δs = distance between velocity or elevation sections
 b_i = average channel width between velocity sections
 h_i = average channel depth between elevation points
 A_i = cross-sectional area evaluated at a section where Q is calculated and primes denote values taken at that time, $t + \Delta t$.

Where junctions of conveying channels occur in the one-dimensional models, the equation of continuity is applied at the junction and it is assumed that there is no significant difference in the elevations of the water surfaces at the entrances to the branching channels (Dronkers, 1964).

It is assumed that A_i is reasonably typical of the section of channel between the elevation points used to calculate Q_i . In principle the empirical adjustment of the friction factor to optimize the agreement between computed and observed elevations tends to compensate for discrepancies, though in the present context, where the depths of the conveying channels are in general much greater than those encountered in rivers, fine adjustment of the friction is not critical to the successful operation of the model.

(c) Second one-dimensional scheme

An alternative finite difference formulation (Dronkers, 1969) of equations (7) and (8) was employed in channels where constrictions and large spatial changes in the cross-sectionally averaged velocities occur (e.g. Seymour Narrows, Sechart Rapids, Fig. 1). Advective acceleration is included in the momentum equation and the distance between sections may be selected arbitrarily to facilitate the schematic representation of a channel using sections of reasonably typical average depth and cross-sectional area or for the numerically convenient spacing of sections near proposed junctions of one- and two-dimensional schemes.

Since the equations are of implicit form, the stability restrictions on the time step occasioned by large depths in certain of the mainland inlets are mitigated. Surface storage and cross-sectional areas are again assumed independent of time.

Using the computational scheme shown in Figure 7, the finite difference forms of equations (7) and (8) are:

$$\frac{1}{\Delta s_i} [Q'_{i+1} - Q'_i] = - \frac{b_i}{2\Delta t} [\zeta'_{i+1} + \zeta'_i - \zeta_{i+1} + \zeta_i] \quad (11)$$

$$\begin{aligned} \frac{g}{\Delta s_i} [\zeta_{i+1} - \zeta_i] = & - \frac{1}{2A_i \Delta t} [(Q'_{i+1} + Q'_i) - Q_{i+1} + Q_i] \\ & - \frac{1}{A_i^2 \Delta s_i} [(Q_{i+1} + Q_i)(Q'_{i+1} - Q'_i)] \\ & + \frac{1}{4A_i^3 \Delta s_i} [(Q_{i+1} + Q_i)(Q'_{i+1} + Q'_i)(A_{i+1} - A_i)] \\ & - \frac{K_i}{4A_i^2(\zeta_i + h_i)} [(Q'_{i+1} + Q'_i)|Q_{i+1} + Q_i|] \end{aligned} \quad (12)$$

where primes denote values taken at time $(t + \Delta t)$ and A_i is the average cross-sectional area over the section.

These equations are solved implicitly using the so-called "double sweep" method (Dronkers, 1969). A detailed account of the method may be found in Mungall (1973).

(d) Junctions between one- and two-dimensional schemes

For reasons discussed below, it is desirable to join the various one-dimensional and two-dimensional schemes. At each junction it is assumed that the width of the entrance to the side channel is the same as the two-dimensional mesh size. The correct cross-sectional area can be obtained by adjusting the depth. If this is significantly different from the average depth of the entrance some error is introduced into the frictional dissipation. This was not found to be significant in the process of overall adjustment of the model.

Considering initially the first one-dimensional scheme, the spatial arrangement of variables in the vicinity of the junction is shown in Figure 8. It will be noted that the section length employed in the side channel is considerably longer than the two-dimensional mesh size. When the one-dimensional or two-dimensional schemes are operated independently, the values $\zeta_1(t+\Delta t)$ or $\zeta_{60}(t+\Delta t)$ are prescribed respectively from observations. When joined, $\zeta_1(t+\Delta t)$ is found from equation (9) where the volume flow entering the one-dimensional scheme $Q_1(t) = V_{97}(t) \Delta z$ and V_{97} is the vertically integrated velocity through the corresponding side of the two-dimensional mesh. $\zeta_{60}(t+\Delta t)$ is then found from the following six-point Lagrangian interpolation using three values of elevation along the normal to the section at the mouth of the channel from each computational scheme.

$$\zeta_{60} = 0.1000 \zeta_{171} - 0.5185 \zeta_{134} + 1.0500 \zeta_{97} + 0.3889 \zeta_1 - 0.222 \zeta_2 + 0.0019 \zeta_3$$

It is worth noting that in the two-dimensional scheme the velocity field is dominated by the component along this normal. When these values of elevations have been determined, the volume transports in each scheme are computed using equations (5), (6) and (10) in the usual manner. In joining the second one-dimensional and the two-dimensional schemes, the need for a spatial interpolation is eliminated by selecting a section length equal to the mesh size at the channel entrance (Fig. 9). In this case $\zeta_1(t+\Delta t)$ is determined from equation (4) in which the substitution

$$V_{i-n\Delta z} = 1/2 [Q_{b-1} + Q_b]$$

has been made.

Conservation of volume of the various junctions was verified by numerical integrations of volume flows over several tidal cycles.

(e) Approximate representation of narrow passes in the two-dimensional scheme

There occur within the region of the two-dimensional scheme a number of navigationally important narrow passes (e.g. Active Pass, Fig. 1) which cannot be adequately represented since the grid size is much larger than the width of the pass. Depending on local coastal configuration and bathymetry, somewhat improved simulations can be achieved in some cases as follows.

If a pass is similar in length to the mesh size, Fig. (10), and the local flows in the adjoining mesh squares are dominantly parallel to the

longitudinal axis through the pass, the change in momentum flux due to the change in width may be approximated for the case of a vertically integrated velocity at one end of the pass by replacing the fourth term on the right hand side of equation (5) with the following expression:

$$\frac{4}{(\Delta x + b)\Delta x} \left[\frac{\Delta x [\bar{U}_i^x(t)]^2}{H_i^x(t) + H_{i+1}^x(t)} - \frac{b [\bar{U}_{i-1}^x(t)]^2}{H_{i-1}^x(t) + H_i^x(t)} \right] \quad (13)$$

To determine the elevation in a mesh adjacent to the pass equation (4) is rearranged using volume transports through the mesh sides, as follows:

$$\zeta_{i+1}(t+\Delta t) = \zeta_{i+1}(t) - \frac{\Delta t}{\Delta x} \left[U_{i+1}(t) - \frac{b}{\Delta x} U_i(t) + V_{i-n+1}(t) - V_{i+1}(t) \right] \quad (14)$$

When the pass is much shorter than a mesh width, a small opening is left in the side of the appropriate mesh. The velocity through the "pass" is then determined using equation (5) with an empirically adjusted friction coefficient to give optimal agreement between computed values and such local observations of tides and streams as may be available for that vicinity. Elevations in meshes adjoining the pass are then found from an equation similar to (14), depending on the orientation of the pass.

(f) Frictional adjustments

A major problem in the development of this model concerns the determination of the proper distribution of frictional dissipation. Ideally, the friction coefficient appropriate to a particular mesh or channel section is determined by direct measurement of surface slopes and velocities entering into a particular finite difference momentum equation. This can be done, at least in principle, for sections in a one-dimensional river model (Dronkers, 1964) but becomes impractical for broad estuaries and coastal seas. In addition to the work of Redfield referred to above, various methods have been employed. Following Taylor (1919); Heisanken (1921), Jeffreys (1921) and Miller (1966) have estimated the frictional dissipation in coastal seas using the net energy flux through boundary openings where elevations and velocities are known. Grace (1936, 1937) divided the Bristol and English Channels into one-dimensional sections in which the elevations were recorded. Using the equation of continuity to obtain the velocities between sections and observed velocity data for one cross-section of the channel, the friction coefficients appropriate to particular sections can be determined. A somewhat similar method has been employed by Filloux (1973).

In the present work, the approach taken involves the empirical fitting of the computed water levels to those obtained from extensive measurements of elevations around the interior of the modelled region. Thus no explicit account is taken of energy lost from the barotropic tides to increase the overall potential energy of the system through vertical mixing, a topic that will be considered in future three-dimensional numerical model studies.

4. DESCRIPTION OF THE MODEL

(a) The Strait of Georgia and Juan de Fuca Strait Scheme

Since the system lies within a relatively small span of latitude a two-dimensional Cartesian coordinate system is employed.

A mesh size of 4 km was selected as providing adequate representation of the cross-channel slopes of the water surface associated with Kelvin wave propagation through the outer and inner Straits (Fig. 11). Coastal boundaries were located along mesh sides where the normal component of velocity is always zero. Due to the irregularities of the coastlines the selection of a particular mesh side to represent part of a coastal boundary is sometimes quite arbitrary. In such cases mesh sides were chosen to ensure conformity between the surface areas of local parts of the model and the actual areas as determined by planimeter from hydrographic charts.

Depths were specified at elevation points and represent the average depth over a grid square. Channels through the San Juan and Gulf Islands were included in this two-dimensional scheme with the exception of four narrow passes where strong tidal streams occur. (Active, Porlier and Thatcher Passes, and Middle Channel, Fig. 1). The individual representations of these passes are discussed below.

The maximum permissible time step yielding stable solutions is obtained from the Courant-Friedrichs-Lewy criterion

$$\Delta t < \frac{\Delta x}{\sqrt{2gh_{\max}}}$$

The resulting time step is 45 seconds. (This time step is used throughout the overall model though longer time steps were generally employed in the individual adjustments of particular one-dimensional models).

Ten openings are located in the boundary of the two-dimensional model. The tidal co-oscillation within the system derives primarily from elevations prescribed across the entrance of Juan de Fuca Strait. At the other nine openings, elevations are determined using five one-dimensional models which are joined to the two-dimensional system.

Initially a constant overall frictional coefficient of .0025 was used in this model. The final values used were only determined after extensive trials of the extended model.

(b) The Puget Sound Scheme

Using the first one-dimensional scheme, the Puget Sound system was simulated by 7 joined channels. Sections where volume transports are calculated are shown in Figure 11. The distance along the median line between sections is 12 km. Elevations are calculated at points located midway between the velocity sections. Volume transports at the closed end of a side-channel are set equal to zero.

The locations of tide gauges from which data were obtained to adjust this model, are shown in Figure 4.

Prior to the addition of this model to the two-dimensional scheme the appropriate overall friction coefficient was determined. The M_2 tidal elevations were prescribed at the entrance and equations (9) and (10) were solved using trial values of the friction coefficients. It was found that good agreement could be obtained between the computed and observed M_2 harmonic constants using a constant coefficient of 0.006 throughout this model.

(c) The Burrard Inlet Scheme

The Burrard Inlet model (Fig. 11) consists of a single channel in which volume transports and elevations at ten cross-sections are calculated using the second one-dimensional scheme. The distances between cross-sections are selected to facilitate specification of average widths and depths for the intervening lengths of channel. Two constrictions occur in this channel known respectively as First and Second Narrows, where strong tidal currents occur (ca. 2-3 m/sec). Tide gauge data are available for 5 locations along the channel as shown in Figure 4.

Initially, M_2 tidal elevations were prescribed at the entrance to the channel. Using an approximate value for the friction coefficient, equations (11) and (12) were then solved using increasingly detailed schematizations of the Narrows until a solution independent of schematization was obtained. (Numerical trials indicate that this is primarily due to the importance of the term $\frac{Q^2}{A^3} \frac{\partial A}{\partial x}$ in the momentum equation.) Trials were then undertaken to determine the values of frictional coefficients along the channel which gave optimal agreement between observed and computed harmonic constants for the various gauge locations. An additional criterion is available in that the phases of the streams through the two Narrows is known from observations. The overall coefficient is .006 while values of .06 are used at the two Narrows.

(d) The Howe Sound Scheme

The Howe Sound model (Fig. 11) consists of six joined one-dimensional channels, two of which connect with the Strait of Georgia. Schematically convenient distances between volume transport sections were employed. Solving equations (11) and (12) frictional dissipation was adjusted in the same manner as that described above for the Burrard Inlet model. It was found that an overall friction coefficient of .006 gave good agreement between the observed and computed M_2 tidal harmonic constants for the gauge location (7811) at the closed end of the system, Figure 4. The amplitudes and phases of the tidal streams in the two entrance channels are strongly sensitive to the elevations prescribed, since the openings are in relatively close proximity, and also to the schematization of these channels.

(e) The Jervis Inlet Scheme

Using schematically convenient distances between volume transport cross-sections, Jervis Inlet was simulated by six joined one-dimensional channels. There are two entrance channels from the Strait of Georgia. Part of the system is separated from the remainder by tidal rapids (Sechlet Rapids, Fig. 1). Indicative of the dynamical effect of these rapids, the change in the M_2 tidal amplitude and phase over their length is about 50% and 60° respectively. Prescribing elevations at the two entrances from local tide gauge data, using equations (11) and (12) the same procedure of optimizing computed values against observations yielded and overall friction coefficient of 0.006. This is increased to .06 in the section simulating the Sechlet Rapids, Figure 1.

(f) The Northern Passages Scheme

By far the most complicated of these one-dimensional models is that which simulates flows entering the northern end of the Strait of Georgia from the open ocean. This system was represented by 25 joined one-dimensional channels and included 94 volume transport sections (Fig. 11). Over this region the observed M_2 tidal amplitudes decrease from 115 cm at the single northern boundary opening to a minimum of 62 cm below Seymour Narrows, increasing again to about 101 cm in the northern Strait of Georgia. Over the same distance the M_2 tidal phase lags increase by 122° . In all there are ten tidal rapids where in some instances, the observed streams can exceed 6 m/sec.

To determine critical features in schematization and the distribution of friction coefficients, the same M_2 tidal elevations were prescribed at each of the four openings onto the Strait of Georgia, based on data from tide gauge #7892, Figure 4. Further M_2 tidal elevations were prescribed at the northern opening based on data from gauge #8233. Using estimated friction coefficients equations (11) and (12) were solved over the course of repeated trials in which lengths of channel most critical to the flow regime were identified, the density of cross-sections along the channel increased where necessary, and the frictional dissipation adjusted, to optimize the agreement between the computed and observed tides. In addition to the tidal data available for the locations shown in Figure 4, the estimated magnitudes and phases of the tidal streams through the more important tidal rapids have been determined from drift pole measurements. In particular, analyses of these data, together with local tidal data, by the Liverpool Observatory and Tidal Institute, permit accurate predictions of times of slack water essential for purposes of navigation. Since the dominant effect of increasing frictional dissipation in one of these narrow passes is to decrease the phase lag of its tidal streams, observed phase information of this kind assists in adjusting friction coefficients. The overall friction coefficient employed in this model was 0.01. Local variations from this value ranged from 0.005 in the deeper parts of Discovery Passage to 0.03 in the smaller channel constrictions.

5. DEVELOPMENT OF THE MODEL

Summarized sequence of development

The final version of the model, Figure 11, was developed over the course of an extensive series of trials. It is not proposed to discuss these in detail but only to present conclusions insofar as they suggested further modifications and trials. On the basis of earlier work with the exploratory multi-channel one-dimensional overall model, it appeared reasonable that the amplitudes and phases of the major semi-diurnal and diurnal constituents in Strait of Georgia could be simulated to within about 5 cm and 5° respectively, with somewhat greater discrepancies in areas where large spatial rates of change in the harmonic constants occur. References below to unsatisfactory results implies a performance well below these standards. The harmonic constants resulting from computed values were determined by Fourier analyses for trials involving a single constituent or harmonic analysis for the case of mixed tides. All phase lags are referred to the phase of the corresponding equilibrium constituent at Greenwich.

Stage 1

The two-dimensional model with eight boundary openings

The most straightforward, yet dynamically plausible, representation of tides in the Juan de Fuca Strait and the Strait of Georgia appeared obtainable from a vertically integrated two-dimensional model in which geostrophic accelerations were included in the momentum equations, frictional dissipation was assumed proportional to the square of the computed velocities and elevations were prescribed at boundary openings to simulate the effects of tides in the adjacent ocean and in the mainland inlets. The two-dimensional grid is that shown in Figure 11.

In this first version the boundary openings included the entrance to Juan de Fuca Strait, seven openings, each the width of a single grid square, representing the main conveying channels into the various mainland inlets. The northern end of the Strait of Georgia was assumed closed except for a single open mesh in the immediate vicinity of gauge #7892, Figure 4, which was intended to allow for tidal fluxes through the Northern Passages. At least one year of tide gauge data had been obtained in the vicinity of each of these boundary openings.

Assuming an overall friction coefficient of 0.0025, a value commonly applied in work on coastal seas, trials were carried out in which the M_2 and K_1 tidal constituents were individually simulated in the model. The agreement obtained between the harmonic constants deriving from the model and those obtained from observations was excellent.

A further trial was undertaken in which a mixed tide consisting of the sum of the M_2 and K_1 elevations, was prescribed at each boundary opening. These computations showed that no significant non-linear interactions occurred in this version of the model. This appeared consistent with the small amplitudes of the overtides and compound tides such as are normally determined in the standard harmonic analysis of one year of tide gauge

observations. This was thought to be due primarily to the relatively large depths which prevail throughout the system. (It will be shown subsequently that significant non-linear interactions do indeed occur.)

At this stage of the investigation, there were insufficient data on the harmonic constants of the tidal streams to verify the computed velocity fields. In general, over much of the model, these fields showed appreciable rotation of the velocity vectors over a tidal cycle. These rotations appeared much influenced by the flows through the various boundary openings.

The magnitudes and phases of the tidal flows through the openings off the Strait of Georgia were, however, quite inconsistent with the anticipations based on results from the earlier exploratory overall one-dimensional model which specifically included inlets associated with these openings. In addition, these flows were superimposed on slowly fluctuating net flows which in some cases were of comparable magnitude with those of tidal period. These irregularities were substantially worse in the case of the K_1 tide than that of the M_2 . In consequence there existed considerable doubt as to the validity of the computed velocity fields which, since the surface tides in the Strait of Georgia might be substantially held correct by the prescribing of elevations at numerous openings along the mainland boundary, would reflect inadequacies in the dynamical assumptions, frictional adjustment and finite difference representation of the topographically complex region of islands between the Strait of Georgia and Juan de Fuca Strait.

Estimated magnitudes of the advective accelerations from the computed velocity fields indicated their dynamical importance in the vicinity of the San Juan islands. It was decided to extend the model and include these terms in the momentum equations.

Stage 2

The two-dimensional scheme with a single one-dimensional scheme and seven boundary openings

The one-dimensional model of the Puget Sound system was selected as the first extension of the existing two-dimensional model for a variety of reasons. The dynamical sensitivity of the model would be much increased if it were required to correctly proportion incoming wave energy between the Strait of Georgia and the largest of the side-channels. It was also desirable that the computed surface tides in the critical region of the San Juan and Gulf Islands should not be favoured by the prescribing of elevations at a nearby boundary opening. Further, the tidal streams in the important area of inner Juan de Fuca Strait are strongly affected by the volume transports through the entrance to Puget Sound System. It was thus appropriate that the magnitude and phase of these transports be verified.

Trials with single constituent tides (M_2 & K_1) in the combined scheme, using overall friction coefficients of .0025 in the two-dimensional and .006 in the one-dimensional, confirmed the results previously obtained when each model was operated independently. In particular the magnitudes and phases of tidal volume exchanges between Puget Sound and Juan de Fuca Strait were in

excellent agreement with those obtained when the elevations had been prescribed at that opening.

Illustrative of the results obtained at this stage of the model development, Table I shows comparisons of the observed M_2 and K_1 harmonic constants with those obtained from the model for representative locations. It is evident that an excellent simulation of the tidal constants in the Strait of Georgia and Puget Sound system had been achieved.

Advective accelerations were now included in the two-dimensional momentum equations (5) and (6) and a further M_2 trial was undertaken. Though no significant overtides were apparent in analyses of the computed elevations, it was found that at all locations in the Puget Sound System there occurred an increase in mean sea level of 5 cm. It was also noted that appreciable residual currents were now apparent in regions of the two-dimensional model where large speeds and changes in direction of velocity occurred.

TABLE I

Comparisons of the dominant semi-diurnal (M_2) and diurnal (K_1) amplitudes (cm) and phases from observations at representative locations with those obtained from the model when a single constituent tide is prescribed at the seven boundary openings of the early version of the model with one side channel.

Location	Gauge Number	M_2				K_1			
		Obs.		Calc.		Obs.		Calc.	
		H	g	H	g	H	g	H	g
Port Angeles	7060	52	75	49	68	66	141	62	142
Pedder Bay	7080	34	76	33	69	63	149	63	149
Finnerty Cove	7140	45	126	42	122	71	157	69	156
Friday Harbour	7240	56	140	58	138	76	161	76	159
Fulford Harbour	7330	59	141	66	141	76	159	77	163
Tumbo Channel	7510	73	159	81	154	81	167	83	164
Ferndale	7564	72	152	79	148	80	163	83	161
Tsawwassen	7590	82	156	86	153	85	164	85	164
Winchelsea	7935	95	161	96	159	88	166	88	167
Little River	7993	99	161	101	160	90	167	90	167
Seattle	7180	107	140	108	141	83	157	86	156

Stage 3

The two-dimensional scheme with four one-dimensional schemes and two boundary openings

There remained, however, the anomalous flows through the various boundary openings in the simulated mainland coast of the Strait of Georgia. To eliminate these and further extend the scope of the model, the one-dimensional Burrard Inlet and Howe Sound and Jervis Inlet schemes were now included in the overall model. The northern end of the Strait of Georgia was assumed closed except, as before, for a single boundary opening, in the immediate vicinity of tide gauge #7892, Figure 4.

An initial trial of this version of the model, in which the value of the overall friction coefficient in the two-dimensional model was .0025 and the M_2 tidal elevations were prescribed at the two remaining boundary openings differences between observed and computed amplitudes and phases throughout the model which were well below expectations. Further trials in which the distribution of frictional dissipation over the model was varied clearly showed the impossibility of obtaining a satisfactory simulation of the M_2 tide in this manner and that there still remained a fundamental inadequacy in the present formulation of the model. This error now appeared to be associated with the reflection problem at the northern end of the Strait of Georgia. Since there existed no way in which the strong tidal flows through the single fictitious boundary opening at the northern end of the Strait of Georgia could be verified, it was decided to solve the reflection problem by simulating the tides in the complex of passages leading northward through Johnstone Strait to the open sea and in the contiguous mainland inlets. Arrangements were made for current meter observations to verify the magnitude and phase of volume transports through the single northern opening to this further system of one-dimensional channels. Thus, if a viable solution to the reflection problem could be achieved either a satisfactory simulation would result or further effort could be concentrated on the problem of wave transmission through the San Juan Islands.

Stage 4

The two-dimensional scheme with five one-dimensional schemes and two boundary openings

Initial trials of the complete model as illustrated in Figure 11 were carried out using an overall friction coefficient in the two-dimensional model of 0.0025 and with M_2 tidal elevations prescribed at the two boundary openings. Again, results were obtained which were well below expectation. This problem could not be resolved by adjusting frictional dissipation in the model since improved phases would be accompanied by disimproved amplitudes. Attention was thus directed to the wave transmission problem between Juan de Fuca Strait and the Strait of Georgia.

Firstly, by introducing perturbations of the M_2 tidal flows through the Northern Passages it was shown that there resulted no excessive changes in the harmonic constants in the amphidromic region in the inner part of Juan de Fuca Strait. Current meter observations over a cross-section of Johnstone

Strait near the northern boundary of the model had become available and confirmed remarkably well the computed amplitude and phase of the M_2 tidal volume flux. Thus, in the present context, the reflection problem was considered solved.

The basic elements considered in the transmission problem concerned the relative significance of numerical approximations (channel depths, coastal boundaries) and frictional adjustments on tidal propagation through the main (Haro and Rosario Straits) and secondary (Active, Porlier, Thatcher Passes and Middle Channel, Fig. 1) conveying channels to the overall tidal regime. Of the two main channels, the computed tidal volume fluxes through Haro Strait are roughly three times the magnitude of those through Rosario Strait. Accordingly, the next step was to show that the secondary conveying channels did not significantly influence the tidal regime in the Strait of Georgia and the Puget Sound system. This was done by comparing the M_2 tidal harmonic constants obtained from a simulation in which these channels were assumed closed with those deriving from the normal case when they are open.

As noted previously, considerable ambiguity exists in the delineation of the coastal boundaries in the San Juan Island region of the model due to the coarseness of the mesh size. Subject to conservation of surface area, a trial was carried out using a plausible but different arrangement of coastal boundaries. This was found to have a negligible effect on the computed harmonic constants in the Strait of Georgia and in Puget Sound.

Trials were then carried out to determine the relative effects of adjusting the assigned depths and frictional dissipation in Haro and Rosario Straits. It was in the course of these trials that the key to concomitant improvements in the computed M_2 amplitudes and phases of both the major tidal storage areas of the model, the Strait of Georgia and the Puget Sound system, was determined. As anticipated, such adjustments in Haro Strait have a much greater influence than when applied to Rosario Strait. This is consistent with the larger volume flow, more complex bottom topography and greater channel length than in the case of the former.

The dominant effect of increasing frictional dissipation in Haro Strait is to increase the M_2 tidal amplitude in Puget Sound with a corresponding, though smaller, reduction in amplitude in the Strait of Georgia. This is qualitatively consistent with the simple theory of superposed damped incoming and reflected waves in a gulf closed at one end. The important point, however, is that the associated changes in the tidal phase lag are relatively small.

If a trial is conducted in which depths are assigned such that there is a contiguous line of meshes corresponding to the maximum depths along Haro Strait and Boundary Passage, the dominant effects are to significantly increase the M_2 tidal phase lag in the Puget Sound system, moderately reduce the phase lag but significantly increase the amplitude in the Strait of Georgia.

These adjustments of friction and depth permitted a simulation of the M_2 tide fully consistent with earlier expectations. The overall coefficient of friction in the two-dimensional model was 0.006. This was increased to .06 in Haro Strait.

Using the same configuration of frictional coefficients and depths, a trial was now undertaken in which the K_1 elevations were prescribed at the open boundaries. The observed and computed tidal amplitudes were in good agreement but throughout the Strait of Georgia, the computed phase lags were too small by about 15° . Such results could not be improved by any adjustment of frictional dissipation, since any improvement in phase must be accompanied by a disimprovement in amplitude.

Stage 5

Frictional dissipation and tidal constituent interactions

Three possibilities were now considered as contributing, at least in part, to the discrepancy between the observed and computed phases of the K_1 tidal constituent in the Strait of Georgia. Firstly, the optimization of the model to simulate the M_2 tide took no account of direct gravitational forcing. In the present case it appeared possible that such forces, though small, might account for some part of this phase error in the K_1 tidal simulation. Secondly, non-linear interactions between the dominant tidal constituents could also contribute to the error. Though the evidence of such interactions in the reported tidal constants appeared slight, the frictional dissipation had been greatly increased and the advective acceleration terms included in the momentum equations over an extensive area of the model since the earlier attempt to find evidence of such interactions in the first version of the two-dimensional model. Thirdly, there exists evidence, at least over part of the year, of the existence of internal tides in both the Strait of Georgia (Chang and Pond, 1976) and Juan de Fuca (Crean and Miyake, 1976). Thus energy might be lost at certain frequencies from the barotropic tides in a manner which could not be represented in the model. The third possibility would logically introduce the need for a three-dimensional model.

Trials were carried out in which the effect of direct gravitational forcing on the simulation of a K_1 tidal co-oscillation in the model were assessed. Since the modelled sea is of limited extent the time-varying components of the K_1 constituent tide generating force were assumed independent of spatial variation over the sea. Normally these force components are available expressed in spherical polar coordinates (Doodson & Warburg, 1941, Proudman, 1953). The components were thus transformed to a Cartesian coordinate system, then rotated to conform with the axes of the model to give the gradients of $\bar{\tau}$ to be introduced into the finite difference formulations of equations (2) and (3). Appropriate corrections were made for the earth's crustal displacements due to the tide generating forces. The K_1 tidal elevations were prescribed on the open boundaries in the usual manner. These trials showed that the introduction of direct gravitational forcing produced no significant change (<1 cm and $<1^\circ$) in the harmonic constants of the K_1 tidal constituent.

Attention was now directed to the possibility of non-linear interactions. To obtain the best possible simulation of mixed tides in the system, the elevations prescribed at each open boundary opening were the sum of the full 61 tidal constituents normally available from the standard one-year analyses of the Marine Environmental Data Service. Using the same configuration of depths and friction which gave optimal results for the M_2 tide trials of mixed tides were undertaken in the model.

It became evident that a reduction in frictional dissipation was necessary. However, the important point was that, in contrast to earlier experience with the M_2 constituent, simultaneous improvements in the tide heights and times of high water in both the Strait of Georgia and the Puget Sound systems could now be effected by a straightforward adjustment of frictional dissipation. Using an overall coefficient of 0.003 in the two-dimensional model and 0.03 in Haro Strait, a one-week sequence of mixed tides was simulated. The agreement obtained between the computed and observed tidal elevations in the Strait of Georgia and in the Puget Sound system was excellent. Further comparisons of observed and predicted tidal streams in Juan de Fuca Strait and the Strait of Georgia yielded similar accord. Some of these results characterizing the performance of the model have been reported (Crean, 1976).

The quality of the results suggested that the major discrepancy in the phase of the computed K_1 tide component in earlier individual simulations was no longer significant, a possible consequence of non-linear interaction. To determine whether such was the case a trial was conducted in which the sum of the M_2 and K_1 constituent tides was prescribed on the open boundaries. The results obtained from this trial may be conveniently summarized by comparing the observed and computed M_2 and K_1 harmonic constants at Point Atkinson as derived respectively from individual and combined simulations of these constituents (Table II). The harmonic analyses in the mixed tidal case employed 48 hours of computed tidal elevations. This record length is insufficient to separate from the K_1 tide the O_1 diurnal constituent brought about by the interaction of M_2 and K_1 constituents. The marked change in phase of the K_1 tide is however confirmed by subsequent analyses of elevations obtained from a simulated 30 days of mixed tides.

It is evident from these results that, at least in the case of these constituents, a non-linear interaction significant to the successful operation of the model, does indeed occur.

It remained to determine whether friction or advective terms in the momentum equations were primarily responsible for this interaction. Advective acceleration terms were removed from the momentum equations and the trial was continued. The results obtained differed insignificantly from those presented in Table II. It was thus concluded that the primary source of this interaction lay in the friction terms.

TABLE II

The amplitudes (cm) and phases of the M_2 and K_1 constituents at Point Atkinson (7795) obtained from the final configuration of the model adjusted to simulate mixed tides when prescribed first individually and then in combination on the open boundaries.

Prescribed tide on open boundaries of the model	M_2		K_1	
	H	g	H	g
M_2	99	168		
K_1			99	148
$M_2 + K_1$	95	164	88	166
From observations	93	158	87	165

6. THE SIMULATION OF MIXED TIDES

The primary aim of this study concerns the reliable prediction of mixed tidal streams throughout the modelled region. The final trial of the model thus consisted in the simulation of tides over a 30-day period for which extensive current meter data were available to check the computed tidal streams. In addition to providing a representative sequence of tides, such a sequence of computed values permits reasonable estimates of the harmonic constants of the major tidal constituents to be made. Unfortunately, however, the synodic period (183 days) of the K_1 and P_1 constituents is considerably longer than the computed records and these constants must be inferred from the amplitude and phase relations of these constituents as determined from longer periods of gauge data for the appropriate locations (Dronkers, 1964). In principle, of course, the run could have been extended to provide data sequences of sufficient length. The high computational cost does not appear to warrant this within the context of the present study.

The period selected for this trial extended from 12 March - 12 April (Julian days 72-100) 1973, when the most comprehensive set of simultaneous data yet available were obtained using moored Aanderaa current meters. The elevations prescribed on the open boundaries of the model were predicted on the basis of 61 harmonic constituents. The friction coefficients and depths conformed to those used in the earlier one-week trial of mixed tides.

Comparisons of the tidal elevations as predicted on the basis of harmonic constants with those computed by the model for some typical locations are shown in Figure 12. The excellent agreement obtained at Little River (7993) and Point Atkinson (7795), Figure 4, is representative of the performance of the model throughout the Strait of Georgia. The results shown for Fulford Harbour (7140) are further illustrative of a similar standard of performance in the complex region of the San Juan Islands. The results obtained for Seattle are typical for the Puget Sound system.

Considering now the tidal current measurements in Juan de Fuca Strait, Figure 13 shows comparisons of the observed tidal streams for station 113, at

depths 15 m, 50 m, 100 m and also at 6 m above the sea bottom, with the corresponding vertically averaged velocity computed by the model. (It will be noted that the abbreviated current meter record at station 113 obtained near the sea bottom has been supplemented by data from the neighbouring station 112. The profiles of tidal currents as evidenced by these data differed insignificantly at stations 112 and 113 as did the corresponding vertically averaged velocities obtained from the model.) Over the period included by these velocity records marked residual flows were occurring in Juan de Fuca Strait associated with the movement of low salinity near-surface waters seaward and a compensatory intrusion of higher salinity water at depth. To facilitate comparison with the results from the model, the non-tidal part of the original records was eliminated by subtracting out the the residuals remaining after the tidal part of the signal had been suppressed by an $A_{24}^2 \cdot A_{25}/24^2 \cdot 25$ filter (Godin, 1972). The current meter records were resolved into components conforming with the U and V directions of the model grid. In this part of Juan de Fuca Strait these U and V components may be regarded respectively as the cross-channel and longitudinal components of velocity.

The excellent agreement shown by the observed and computed longitudinal components of velocity was typical of all the records obtained over this cross-section. No significant differences in phase occur, even near the bottom (157-160 m) where it might reasonably have been anticipated that the influence of friction would lead to a somewhat earlier turning of the streams. At 15, 50 and 100 m, the ranges contained by observed velocity extrema tend to be somewhat larger than those given by the model. Near the bottom, however, the values given by the model are larger than those observed. This accords reasonably well with the vertical distribution of velocity normally obtained in open channel flow despite the presence of vertical stratification. Further improvement in the comparisons between these velocities could be obtained by inferring depth dependent velocities from an empirically determined profile and the depth-mean tidal stream velocities obtained from the model (Dronkers, 1964).

The transverse (U) components of velocity are clearly subject to additional modes of oscillation not present in the barotropic model. As noted above the stratification in the Strait of Juan de Fuca, which resembles that of a deep partially mixed estuary, changed persistently over the course of the observations. At 100 m depth the observed and computed cross-channel velocities agree well with regard to phase. There appears, however, to be a marked increase in the range of observed velocities when the diurnal inequality is minimal. It is of interest to note that over days 79-81 these increased ranges at 100 m are accompanied by an approximate phase reversal of the velocities at 15 m. A concomitant sequence of (STD) casts in the vicinity of the line of current meters over this period showed changes in the depths of isopycnals of roughly semi-diurnal period and displacements of up to 40 m.

Over the same period, a single Aanderaa current meter was moored at a depth of 18 m on the median line of the channel south of Race Rocks (Station 001, Fig. 4.). Primarily due to the smaller cross-sectional area of the Strait, the velocities are considerably larger than at the previous location. Comparisons of the observed and computed velocities are shown in Fig. 14. The computed U and V components do not conform to the longitudinal

and cross-channel directions. In general the agreement between the phases of the observed and computed velocity series is excellent. As anticipated, the observed velocity ranges are somewhat larger than the computed vertically averaged velocities. Again, the use of an empirically determined correction generally descriptive of open channel flow would substantially improve the agreement.

A further array of Aanderaa current meters, Line 1, Figure 4, was moored over a cross-section of Johnstone Strait near the northern boundary of the model where an average velocity over a cross-section is calculated. This computed velocity was corrected for the slightly different cross-sectional area of the location where the current meters were moored. The computed velocities (Fig. 15) along this channel indicate a negligible change in the phase of the tidal streams between the two locations. As in the case of the Juan de Fuca Strait section, marked residual seaward flows were observed near the sea surface accompanied by compensatory inward flows near the bottom. Density stratification in this channel is minimal. Comparisons of the observed tidal streams at depths 15 m, 75 m and 225 m for station 3 centrally located in the Strait are shown in Figure 14. At all three depths the phasing of the observed streams is in excellent agreement with that obtained from the model. Excellent agreement is also obtained between the observed and computed velocity ranges at 15 and 75 m. At 225 m, the observed velocity ranges are appreciably larger than those computed. Similar results were obtained from other current meters in the array. It has been suggested that a longitudinal internal seiche motion associated with the weak density stratification and excited by the surface tide may be responsible for this anomalous vertical distribution of velocity (Thomson, 1976). Subject to this reservation, however, it would appear that velocities computed at the northern boundary of the model are in good accord with observations.

Indicative of the capacity of the model to predict the fast tidal streams in one of the narrow passes, Figure 16 shows the tidal velocities computed by the model for Seymour Narrows with those predicted by the Canadian Hydrographic Service. Again it will be found that the observed and computed values are in good agreement.

In referring to the early one-dimensional numerical model trials, it was noted that a realistic anticipation of performance for a barotropic numerical model of the system would involve simulating the tidal harmonic constants to within about 5 cm and 5° in the Strait of Georgia and Puget Sound. It is thus of interest to compare the constants deriving from harmonic analyses of the computed elevations over a 30-day period for some typical locations with those observed (Table III). In these analyses, the separation of pairs of constituents having synodic periods in excess of 30 days has been done by inference based on known amplitude and phase relationships from one year of gauge observations at the corresponding places.

When the harmonic analyses are applied to 30-day records obtained from the tide gauges and to the same 30 days of computed elevations for corresponding locations in the model, excellent agreement is obtained between the respective harmonic constants for the six main constituents. This implies that the "contamination" of these constituents by others which cannot be separated out on the basis of this record length is essentially similar in both the model and in nature.

TABLE III

Comparisons of the amplitudes (cm) and phases of the six dominant constituents as obtained from observations at four typical locations with those derived from harmonic analyses of 30 days of computed elevations in the model. (The K_1 and P_1 constituents have been inferred using amplitudes and phase relationships based on 1 year of observations).

	Little River 7993				Point Atkinson 7795				Fulford Harbour 7730				Seattle 7180			
	Obs.		Calc.		Obs.		Calc.		Obs.		Calc.		Obs.		Calc.	
M ₂	99	161	101	167	93	158	92	164	59	141	59	150	107	140	101	140
S ₂	25	182	26	184	23	179	23	181	14	157	14	163	26	158	24	158
N ₂	22	138	22	140	19	135	20	137	13	116	13	119	21	113	21	117
K ₁	90	167	91	171	87	165	86	170	76	159	76	165	83	157	82	161
O ₁	49	152	51	151	57	151	49	150	42	146	45	146	46	144	44	140
P ₁	29	166	30	172	27	165	29	172	24	158	25	166	25	155	25	159

In the case of the Strait of Georgia and Puget Sound (Little River, Point Atkinson and Seattle) agreement has been obtained within 6 cm and 6°. At Fulford Harbour there is a phase discrepancy between the observed and calculated phases of the M₂ tide amounting to 9°. This is, however, a channel where the tidal phase changes rapidly with distance.

On the results of sensitivity trials described above in Stage 4 of the development of the model, the discrepancies between the computed and observed M₂ tidal amplitudes and phases in the Strait of Georgia and Puget Sound could probably be reduced by manipulating the frictional dissipation and depths in Haro Strait. It appeared, however, that further adjustments of the model were scarcely warranted and that a finer grid schematization of the San Juan and Gulf Island region was required.

7. DESCRIPTION OF MODEL GF3

To obtain a better resolution of the tidal streams in Juan de Fuca Strait, the San Juan and Gulf Island passages, and the southern part of the Strait of Georgia, a smaller mesh (2 km) model was developed taking its prescribed boundary elevations from the overall model (GF2) described above.

The scheme used in the model fine grid (2 km mesh) is shown in Fig. 17 where each square corresponds to the four sides of a mesh as illustrated in Fig. 5. Coastlines are simulated by mesh sides where the normal component of velocity is set equal to zero. Similarly, narrow islands or land barriers are depicted by closed mesh sides within the interior of the sea. In some instances ambiguities exist as to which side of a mesh best represents a part of the coastline. Such ambiguities are resolved by ensuring that regional surface areas in the model accord with those obtained from hydrographic charts.

The average depth over a mesh is specified at each elevation point within the interior of the sea. In regions where banks may be uncovered at low water, notably near the mouth of the Fraser River, an arbitrary depth of 5 m has been assigned. This is considered adequate at this stage of development of the model. Any realistic numerical simulation of flows over the shallow banks near the Fraser River mouth must also take account of the river discharge and its variations, and also the motion of the salt wedge.

A number of passes are included within the modelled regions such as, for example, those leading from the region of the Gulf Islands into the Strait of Georgia. In some cases these have been simulated by one-dimensional channels inserted into the two-dimensional scheme. Earlier work with the overall tidal model showed that the comparatively small flows through these passes are not critical to the proper simulation of the tidal streams in the main conveying channels of the system, though they do possess considerable local importance. These features of the model will not be discussed here. Detailed adjustments of the flows through these passes will be undertaken when pertinent field observations, presently planned, are completed.

The value of the friction coefficient used in the Strait of Georgia and in Juan de Fuca Strait is 0.003. In the region of the channels between the

San Juan and Gulf Islands this value was increased to 0.03. These values accord with those used in the overall model (GF2) described above.

When the model is operated in the absence of lateral stresses small grid-scale fluctuations in the directions of velocity vectors ("noodling") occur in regions where the advective accelerations are important. The value of the horizontal eddy coefficient used to eliminate this feature of the model was $10^6 \text{ cm}^2/\text{sec}$, which, in view of the mesh size and timestep, implies a relatively small degree of lateral averaging (Kuipers and Vreugdenhil, 1973). The problem of numerically evaluating second derivatives near coastal boundaries was avoided by setting the coefficient equal to zero in such circumstances. The implied assumption is that generally the bottom stress acting on the water in a relatively shallow mesh contiguous to the coast is much greater than the lateral stress.

The maximum permissible time step yielding stable solutions, obtained from the Courant-Friedrichs-Lewy criterion, is 23 secs.

Elevations were prescribed along the three open boundaries of the model shown by the lines, AB, C, DE in Fig. 17, by interpolation of values from the overall model of the system which used a 4 km mesh. The boundary conditions in the case of the latter were predicted tides based on 61 harmonic constituents. As noted above, earlier attempts to operate part of the 4 km mesh model by prescribing elevations at these locations failed, firstly, because of the insufficient sensitivity of the model to permit proper adjustments of the frictional dissipation by optimising computed and observed elevations around the interior of the modelled sea, and secondly, because of the inability of tide gauges at either end of a long open boundary to adequately resolve the slopes of the water surface required to balance the geostrophic acceleration of the tidal streams moving normal to the open boundary. To overcome these difficulties in the present case, the distribution of frictional dissipation and the elevations of the water surface presented at open boundaries, were taken from the original overall model of the system. Subsequent comparisons of the velocity vector fields obtained at comparable instants from the two models showed excellent agreement except where modified by the changed representation of the coastal boundaries.

8. SIMULATION OF MIXED TIDES IN MODEL GF3

Illustrative of the performance of the model, some comparisons of computed elevations with those predicted on the basis of 61 harmonic tidal constituents over a seven-day period are now presented for the representative locations around the interior of the modelled sea, Fig. 4. At Tumbo Channel (7510) and Ferndale (7564) the excellent agreement obtained is typical of the results obtained throughout that part of the Strait of Georgia included within the model, Fig. 18.

The most difficult area of the model in which to obtain good agreement is in the region of Haro Strait and in the degenerate semi-diurnal amphidromic region in the inner part of Juan de Fuca Strait, regions characterized by rapid changes in the spatial distribution of the harmonic constants, Fig 3.

A comparison of the predicted elevations with those computed by the model for Fulford Harbour (7330) shows good agreement though the curves from the model show a small phase lead throughout the record and there exists a modest discrepancy in the tidal ranges. More detailed adjustments of frictional dissipation in this part of the model, including that in the passes leading into the Strait of Georgia from the region of the Gulf Islands could probably improve the comparison. Such adjustments are presently considered of secondary importance to the successful reproduction of the tidal streams in the main conveying channels of the system and will not be attempted until the acquisition of further field data.

The Pedder Bay (7080) gauge is located near the semi-diurnal amplitude minimum. The agreement obtained is good though there is evidence of more accentuated higher tidal harmonics in the values from the model.

The primary object of this numerical study concerns the reliable prediction of the barotropic tidal streams.

During the interval of time selected for this trial of the model an extensive array of current meters was in position over a cross-section of Juan de Fuca Strait. The location of the moorings are shown in Fig. 4, line 11. Four Aanderaa recording current meters were suspended at each mooring in the deepest part of the channel. Comparisons representative of the agreement between the vertically averaged velocities predicted by the model and measured tidal velocities in this section are shown in Fig. 19. The measured velocities are resolved in the direction of the model axes and at this location may be considered as the longitudinal and cross-channel components of flow. (For purposes of the comparison the low frequency non-tidal part of the measured velocities has been removed by first suppressing the tidal signal by means of an $A_{24}.A_{25}/24.25$ filter (Godin, 1972), and then subtracting the residual from the original record). The comparisons of the observed and predicted longitudinal V components show excellent agreement with respect to phase. The range of the observed velocities at 15 m and 50 m is appreciably greater than that predicted by the model. This feature, however, accords well with expectations based on observed profiles in open channel flow. Unfortunately, current meter malfunctions precluded the presentation of a further comparison close to the sea bottom. Such comparisons in the base of the more extended run of the original overall coarse grid model show that indeed at some 5-6 metres off the sea bottom the predicted, vertically-averaged, tidal velocities are considerably greater than those observed.

The smaller measured cross-channel components of velocity show much more variability than those directed longitudinally. Occasional intervals of good agreement between the values predicted by the model and those observed are thought to conform essentially to barotropic tidal oscillations, while the more irregular motions are associated with the intermittent excitation of internal modes. The region where these observations were made is stratified. During the period of the measurements, time series STD observations (Crean and Miyake, 1976) indicated displacements in the depths of isopycnals near the coastlines of up to 40 m which were approximately of semi-diurnal period.

In addition to the above, comparisons were also made between the elevations and velocities obtained from the overall coarse grid (GF2) and limited-area fine grid (GF3) models respectively. No significant differences were noted between the results obtained from each model except in those parts of the velocity fields which are strongly influenced by local coastal boundaries.

Fig. 20 shows the tidal elevations and the longitudinal component of the tidal streams, as obtained from the model for the period 1200, 15 March - 1600, 16 March 1973, at a central location in the Strait of Georgia off the mouth of the Fraser River. The range between higher high-water and lower low-water conforms approximately to the range contained between mean higher high-water and mean lower low-water at this location. Corresponding vector plots of the ebb and flood velocity fields when the streams are maximal are shown in Figs. 21 and 22. [Small crosses denote the centre of a mesh where the velocity components, respectively averaged over those obtained from opposite mesh sides, are plotted. When the line denoting the velocity magnitude and direction exceeds 30 cm/sec, the vector is represented by two parallel lines of equal length, and so on for succeeding multiples of that speed, to avoid confusion due to excessive overlapping.] Throughout most of the Juan de Fuca Strait and the Strait of Georgia the tidal streams are essentially rectilinear. In the inner part of Juan de Fuca Strait there occurs some rotatory character to the streams.

The velocity vectors suggest some interesting aspects of the tidal flows near coastal features off the southernmost part of Vancouver Island where large speed and marked changes in flow direction occur. The formation of back eddies changing in location from ebb to flood is clearly evident. Examination of a time sequence of such vector plots indicates that the eddy begins to form when the stream at that particular location has attained sufficient magnitude; the eddy then increases in spatial extent until the stream turns. A further point of interest concerns the difference between strength and direction of the ebb and flood streams in Haro Strait, again emphasizing the importance of local topography. It is of interest to note that these flow features lie along what is probably the primary route followed by water from the Fraser River moving seaward through Juan de Fuca Strait.

In collaboration with Mr. W.S. Huggett of the Canadian Hydrographic Service, a number of field operations have been carried out to check the presence of major eddies occurring in the model. Lines of drogues were placed across the predicted locations of the major eddies. A comparison of one such set of velocities obtained from the model with those actually observed at a comparable stage of the tide is shown in Fig. 23, for the southern end of Haro Strait. The location of these observations is shown in Fig. 24.

The distribution of residual tidal velocities obtained from a 30-day simulation of mixed tides (61 constituents) is shown in Fig. 24. Though some doubt exists concerning the distribution of net flow near the mouth of the Puget Sound system (a boundary opening), the complex pattern of strong residual circulation associated with the large tidal eddies is clearly in evidence. In collaboration with Dr. J. Stronach, Lagrangian tracks of particles within this residual velocity field have been determined.

Verification of the residual circulations associated with one of these eddies was undertaken in collaboration with Dr. R.A. Thomson and Mr. W.S. Huggett. Fig. 25 shows a comparison of observed and computed (GF3) residual velocities using an array of current meters located in the region where the stream from Haro Strait enters the southern part of the Strait of Georgia through Boundary Pass. There is good agreement between the observed residual velocity vectors at different depths and the barotropic tidal residual vectors predicted by the model. Observed residual currents in the region extending from the southern end of Haro Strait to Race Rocks, Fig. 26, (Mojfeld 1979, Holbrook, et al 1980) also show a good agreement with those obtained from the model (GF3).

A number of interesting dynamical inferences may be made from slopes of the water surface as illustrated by three-dimensional plots such as those shown in Fig. 27. In crude terms, all land has been removed above a base plane corresponding to the lowest elevation of the water surface computed in the modelled sea at the particular instant in question. Thus, islands are represented as 'holes' in the sea surface. The intersection of two lines in the sea area corresponds to the centre of a mesh in the model where elevations are calculated while a single line indicates the elevations along a channel represented in the model by an isolated sequence of contiguous meshes. The fields of elevation coincide in time with the ebb and flood velocity fields shown in Figs. 21 and 22 respectively.

As noted above, the proper proportioning of tidal elevations and times of high water between the Strait of Georgia and the Puget Sound system required the introduction of high frictional dissipation in the region of the San Juan Islands. The marked slopes of the water surface through this region are clearly evident. Elevations in the area contained between Vancouver Island and the Gulf Islands are similar to those in the northern part of Haro Strait and can differ markedly from those in the Strait of Georgia. The resulting strong tidal flows through the passes leading into the Strait of Georgia are well known to local navigators.

In the Strait of Georgia, there are cross-channel slopes of the water surface which balance the geostrophic accelerations due to the tidal streams moving parallel to the major axis of the Strait, while in the vicinity of the river mouth local slopes are associated with the motion of water respectively, off, or onto the shallow banks.

A further feature of these diagrams concerns secondary undulations of the water surface in the inner part of Juan de Fuca Strait. These are more strongly in evidence when the streams are running full than at slack water and will be investigated further. At present it appears that they may be associated with depressions in the water surface of some 10-20 cm near the centre of the large eddies near coastal features which have been referred to above.

9. APPLICATIONS OF RESULTS FROM GF2 and GF3

An immediate application of the numerical model results may be found in the preparation of a tidal current atlas in which the computed flood and ebb tidal velocity fields would be presented at hourly intervals over

representative tidal ranges. Provided reservation is made for flow features unresolved by the necessarily crude coastline of the model, good predictions of the tidal streams for the main conveying channels of the system can be obtained. Of special interest is the growth and decay of large scale eddies that occur in particular parts of the system. Significant departures of the near-surface velocities from the vertically averaged tidal velocities will occur when winds are strong and must be empirically corrected for by the navigator.

A problem arises when an additional influence on the near-surface velocities must be considered. Thus the shallow plume of brackish water formed by the Fraser River discharge over the denser underlying saltwater introduces dynamical considerations which are not included in the present models. A further model (GF4), Fig. 17, is being developed specifically to determine motion in this upper layer in collaboration with Dr. J. Stronach.

If the velocity fields have been computed by the model for a sufficient length of time, tidal harmonic constants may be determined for each velocity point. Such constants may then be employed to predict the velocities at any point within the modelled region for times not included within the original sequence of computed tides. Comparisons of velocities computed directly by the model with those predicted using constants from an earlier run of the model for a typical location in Juan de Fuca Strait are shown in Fig. 28. Also included within the figure are predictions with and without the use of known amplitude and phase relationships from long tide gauge records to separate constituents, not otherwise separable using one month of velocities computed by the model. The availability of such constants makes it possible for a shipboard computer, using a standard tidal prediction program and stored constants, to predict the tidal currents in the vicinity of its position, or course alterations required to maintain a desired track in the presence of transverse tidal stream. An additional refinement could be the inclusion of components of surface drift and wind drag on the vessel's superstructure, using winds obtained from the ship's anemometer. In the case of search and rescue operations the net drift of a floating object could be estimated, continuously updated and presented on a small video screen. Uncertainties of initial position could be included to give patterns of probability of the objects location. A further refinement could be the inclusion of relevant hydrographic chart data.

As noted above in connection with the tidal current atlas the necessity of using a grid of square meshes for the computation of the velocity fields results in poor resolution of these fields around coastal features of the same scale as the grid size. Hence it is desirable for local regions of complex coastal topography, where tidal currents are strong, that computations be undertaken using a smaller mesh size. Over large areas such a smaller mesh size would result in prohibitive computing costs. Conditions must be prescribed on the open boundaries of the smaller area model. If these are taken directly from observations, the inherent errors in these observations tend to dominate the small area model (SAM). If, on the other hand, the small area has been reasonably represented in terms of surface area and mean depth in a large overall model, values of elevations and velocities can be spatially interpolated to provide the open boundary conditions for the smaller mesh model, a relatively common practice.

A test of such a small model was carried out for the complex region of strong tidal streams and large eddies located in the region extending from Race Rocks to the southern part of Haro Strait, Fig. 1. An example of the velocity field computed by Dr. J. Stronach using a 1 km mesh model, employing spatially interpolated elevations and velocities from GF3 on its open boundaries, is shown in Fig. 29.

Such small area models could be useful for detailed flow studies in the vicinity of sewer outfalls or coastal construction sites. In this latter instance, however, the location of the open boundaries must be such as to be negligibly influenced by alterations in the flow field brought about by new construction. Typical examples for such applications could be Haro Strait, a complex region of relatively well mixed strong tidal streams, or the Roberts Bank superport construction.

A particularly interesting use of the elevations and velocities computed by these vertically integrated models is being undertaken in collaboration with Dr. J. Stronach. A major concern is the motion of fresh water in the southern part of the Strait of Georgia, which may move southward into the strong tidal mixing area of the San Juan Islands or northward into the Strait of Georgia where the tidal flows are relatively weak. In the latter case, it may eventually move seaward through Johnstone Strait or the accumulated discharge over a lengthy period may be returned southward by the action of winds. The problem is complex. The thickness of the plume is commensurate with the tidal range at the river mouth. There exist extensive areas of drying banks. The intrusion of the salt wedge into the Fraser River is related to the state of the tide and to variations in the strength of the river discharge. There is also the problem of simulating propagating shallow fronts between river water and sea water, or the remains of preceding "plumes", in the Strait. Any attempt to include all these features in a single three-dimensional model would likely, at present, merely proliferate dynamical confusion at heroic expense. The simplest approach of holding promise of simulating the dominant aspects of the shallow upper layer motions in the Strait of Georgia, and which utilizes data from the above barotropic tidal model GF3, will be summarized briefly. The area included by this model (GF4) is shown in Fig. 17.

Consider initially a system in which a lighter layer of fluid is superimposed upon a denser lower layer and in which the motions within each layer may be described by the vertically integrated equations of motion and continuity as applied to a flat rotating sea (Proudman, 1953). In the present case, the top layer is confined to the upper few metres of the water column which is generally some 150 - 300 m deep. Consistent with observations from moored current meters over a cross-section of the Strait near the river mouth, the presence of this upper layer does not significantly effect the motion in the lower part of the water column which to a first approximation is dominated by the barotropic tidal streams. If now that part of the pressure gradient in the lower layer which derives from the presence of the upper layer, is set equal to zero, equations of motion and continuity applying to a single upper layer may be obtained in which the thickness of the layer now replaces the elevation of the water surface as one of the dependent variables, and in which additional terms in the momentum equations effectively induce "buoyant spreading". Empirical terms are also included to approximate entrainment of water from the lower layer into the upper. A

particle of water in this upper layer will experience a pressure gradient associated with those slopes of the water surface giving rise to the barotropic tidal motions in the much thicker lower layer. At any point there will also exist a stress associated with the difference between the upper layer velocity at that point and the barotropic tidal velocity underneath. Both the components of the free surface barotropic tidal slope and underlying velocities are known throughout the Strait from the original tidal model GF3. The addition of a salt conservation equation, with an empirical expression to include entrainment, permits computation of the spatial distributions of upper layer salinity in the southern Strait. These salinities may then be used to approximate the field of density. There is now at hand a simple and relatively inexpensive system which permits numerical experiments in connection with the changing surface slopes of the barotropic tide, varying river discharge, entrainment, mixing, vertical and lateral stresses, wind stresses, field accelerations and the formation of adequate boundary conditions.

An important feature of this approach is the relaxation of the stringent limitation on the time step imposed by the propagation speed of free surface waves in a region where a small mesh size is required and the water is relatively deep. An appropriate program of field measurements, which include STD casts, velocity profiling, drogue tracking, and also the recording of surface velocities at the river mouth has been completed. The numerical model work is presently in progress.

10. PRESENT AND FUTURE WORK

As described above, the eventual object of this work is to quantitatively estimate the changing exchange of fresh and salt water between the Georgia/Fuca systems and the adjacent areas. Two basic processes can be distinguished. The exchange can occur through the seaward movement of brackish water near the surface entraining seawater from below. The replenishment of the latter requires a counterflow of incoming seawater at depth. The deep inflow can however be radically increased if the density of the water at the ocean entrance is increased. Such processes can be radically modified by topography, tides and winds. Thus the vertical stratification within the system varies from the highly stratified fjord type through the partially mixed to the fully mixed type.

The numerical approach to simulating these processes employs the shallow buoyant spreading upper layer model (GF4) referred to above, to model the motion of the shallow Fraser river plume in the southern part of the Strait of Georgia. To simulate deep water movements associated with observed seasonal changes in the density of water at the ocean entrance, two different types of model are being employed.

The first of these models (GF5) is a laterally integrated two-dimensional model (x, z, t dependent) of Juan de Fuca Strait and the Strait of Georgia. The primary object of this model is to determine possible ways of introducing predetermined barotropic tidal velocity fields into the baroclinic calculations to increase the time step and thus reduce computing costs. The basic assumption is that the barotropic tides are not affected by the density-driven flows but that the latter are significantly affected by the former.

The second model (GF6) is the result of a collaboration with Dr. Jan Backhaus of the University of Hamburg and is a three-dimensional version of the earlier two-dimensional model GF2. Three-dimensional numerical models of coastal seas are still in a relatively rudimentary stage of development. Important considerations in this work are the extensive observations available in both the North Sea and the waters between Vancouver Island and the mainland and also the opportunity of testing a numerical scheme, developed over a number of years for North Sea application in a different dynamical context.

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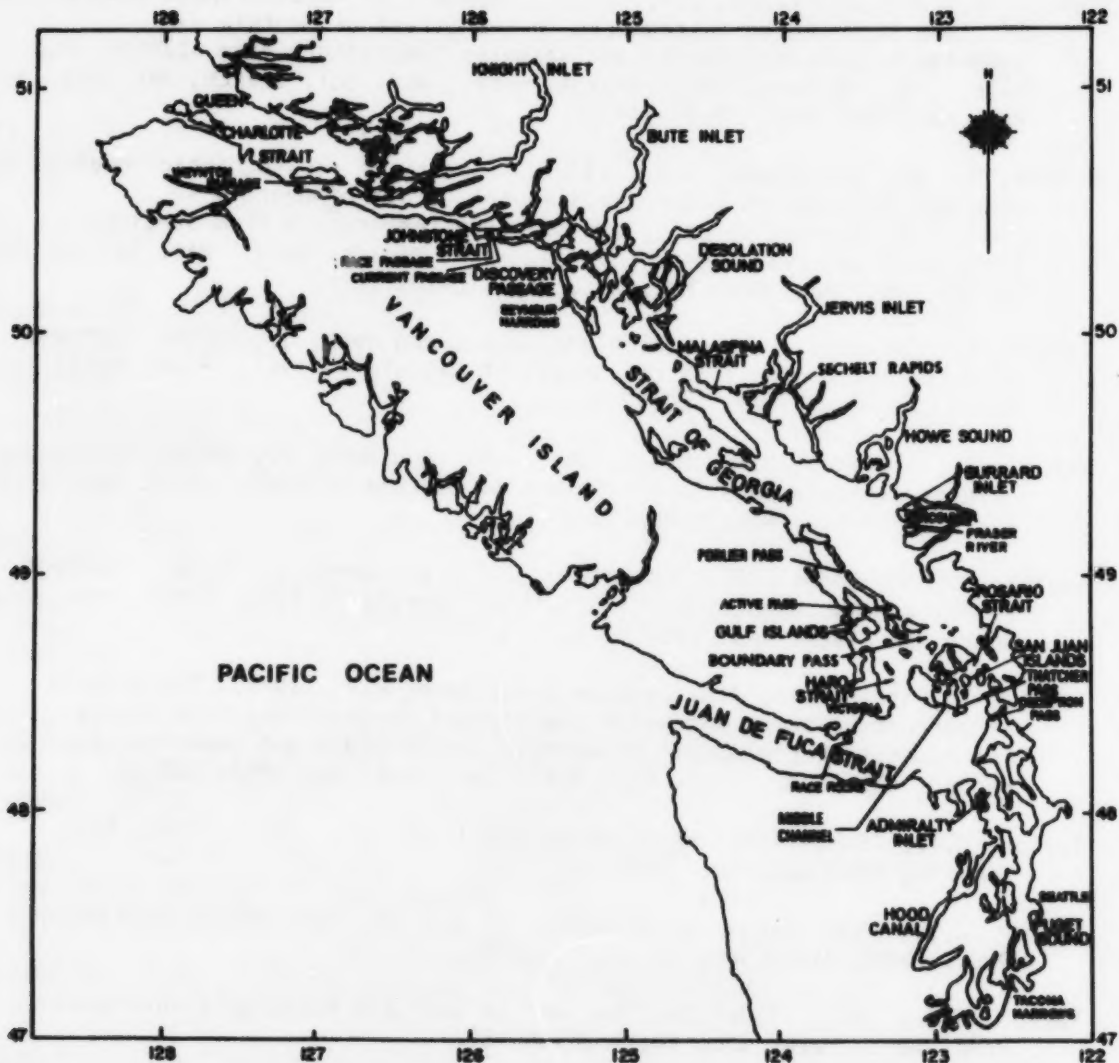


Fig. 1 Map of the waters between Vancouver Island and the mainland coasts of British Columbia and the State of Washington.

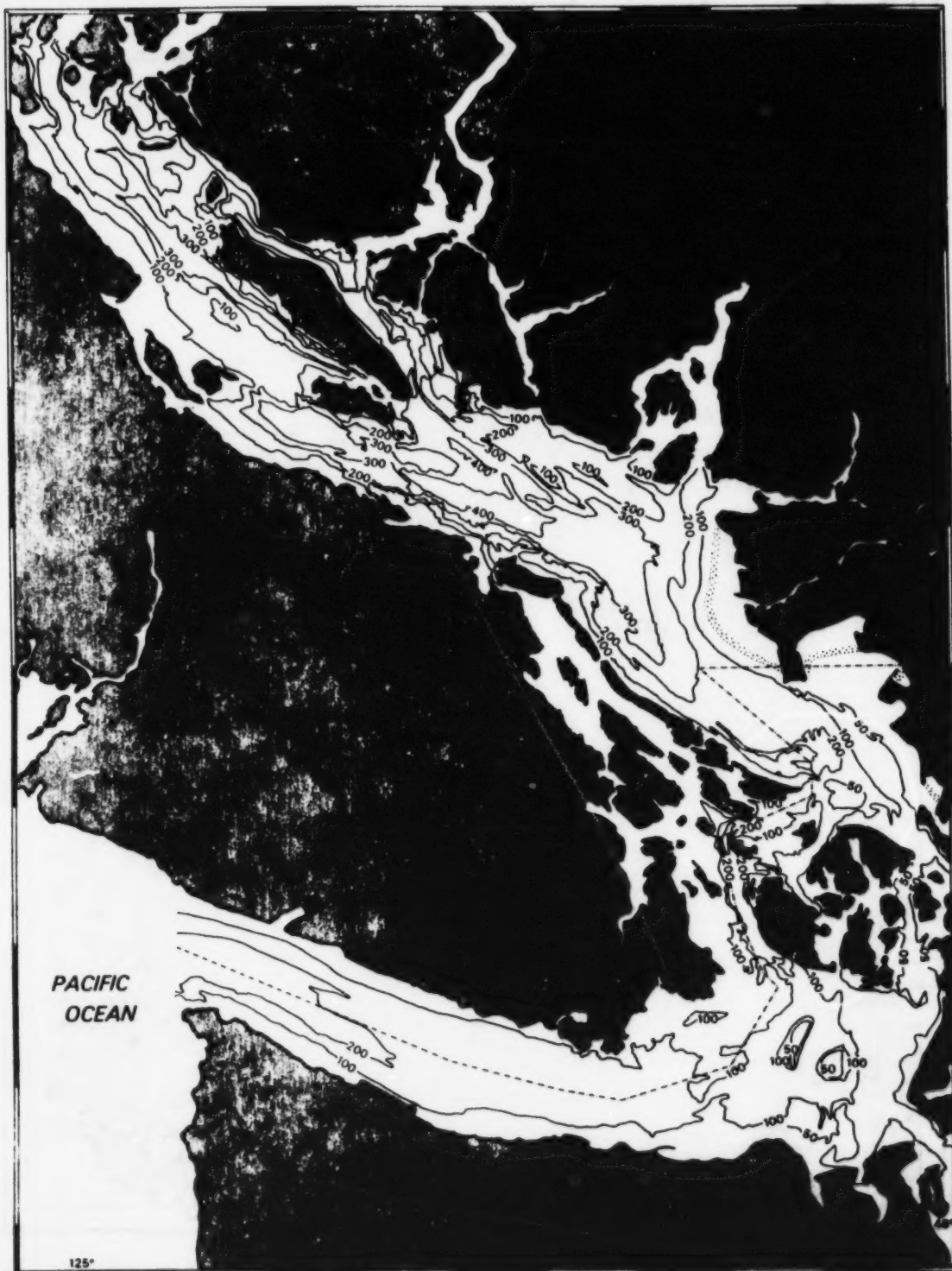


Fig. 2 Bathymetry of Juan de Fuca Strait, Strait of Georgia and San Juan Islands region.

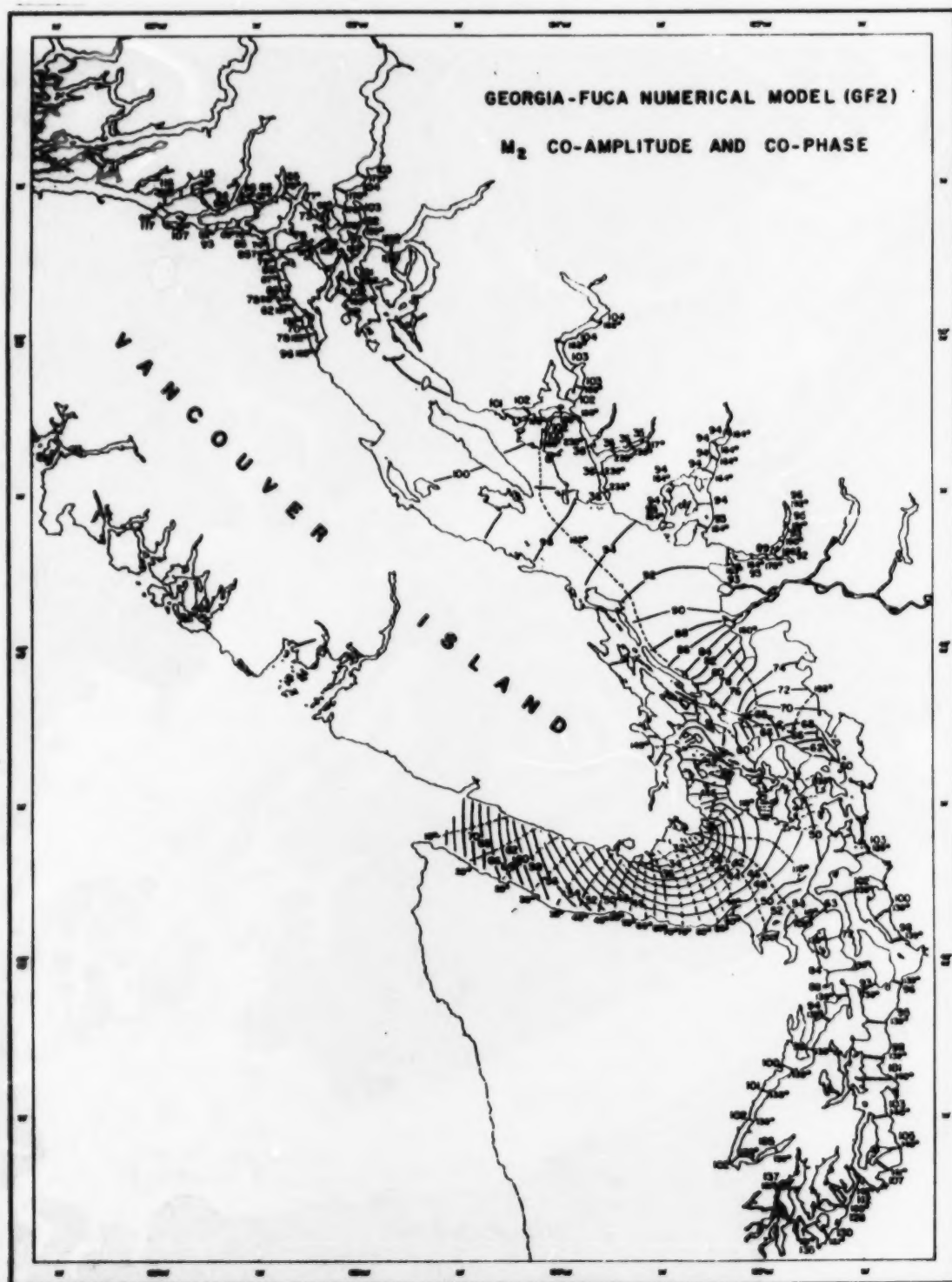
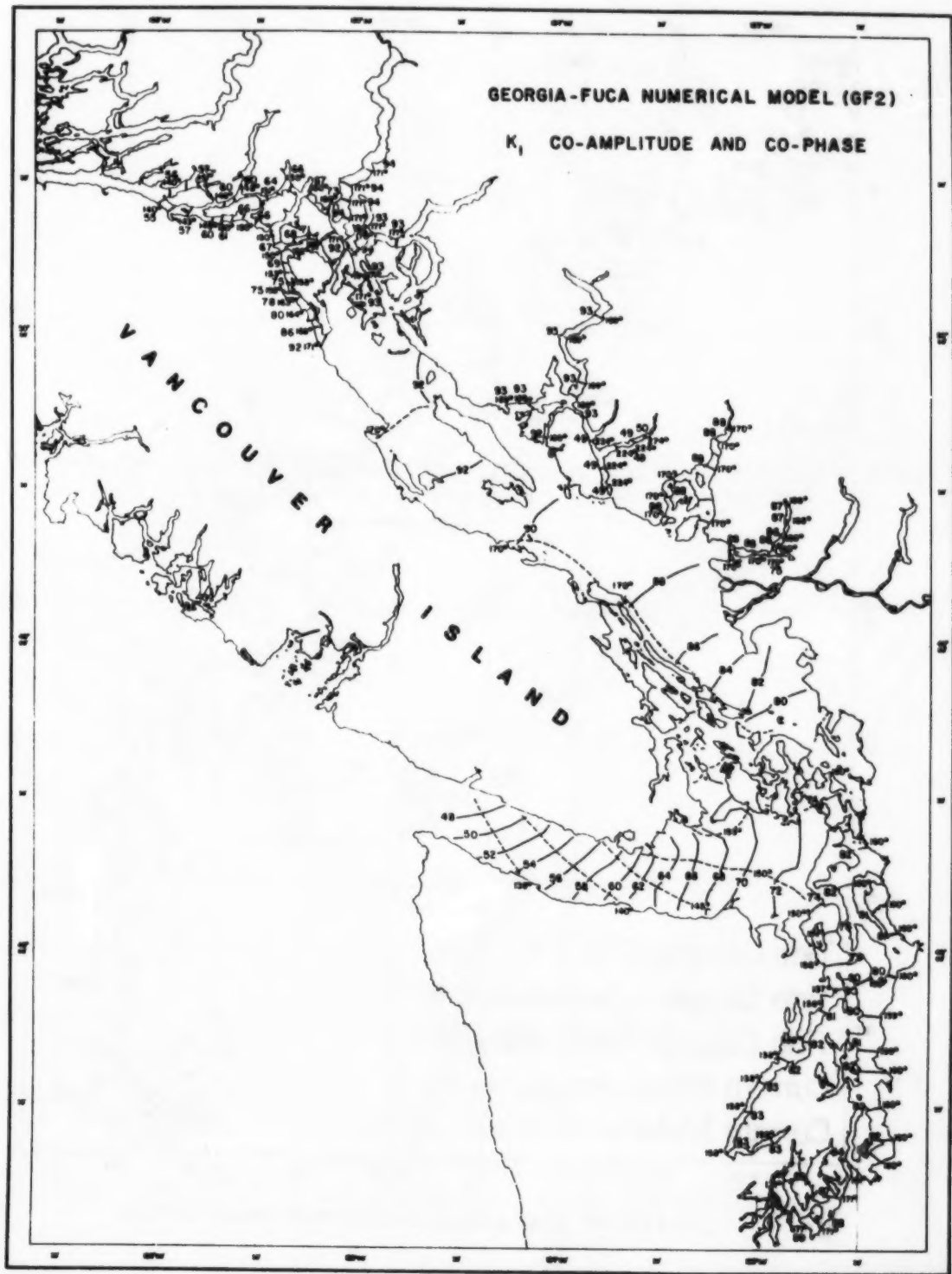


Fig. 3 The co-amplitude and co-phase lines for the a) M₂ and b) K₁ constituents in Juan de Fuca Strait and the Strait of Georgia for the M₂ and K₁ constituents. (These diagrams are based on data computed by an early version of the model but are substantially correct).



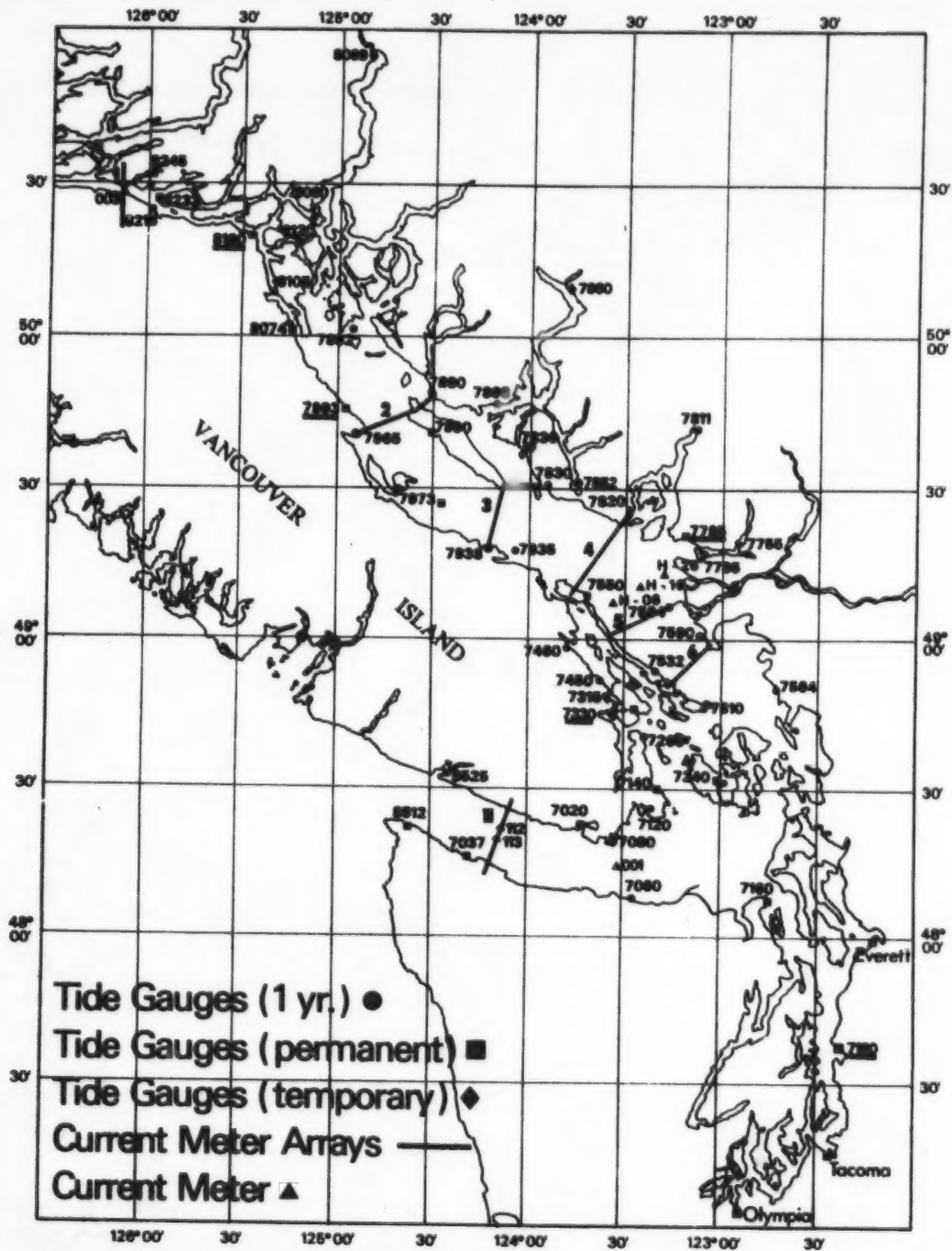


Fig. 4 Location of tide gauges and current meter arrays.

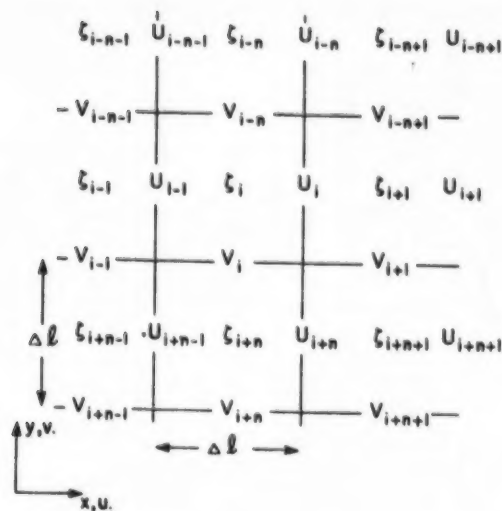


Fig. 5 The two-dimensional grid. The ζ points denote locations where elevations are calculated and depths specified.

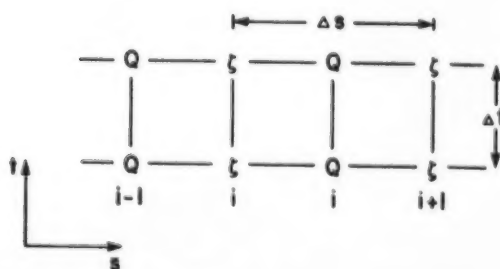


Fig. 6 The first one-dimensional scheme.

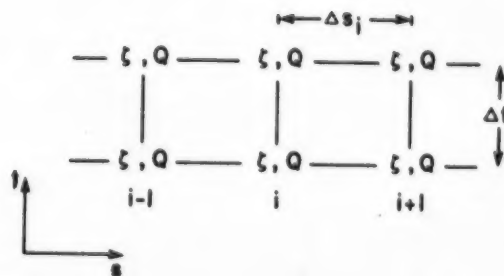


Fig. 7 The second one-dimensional scheme.

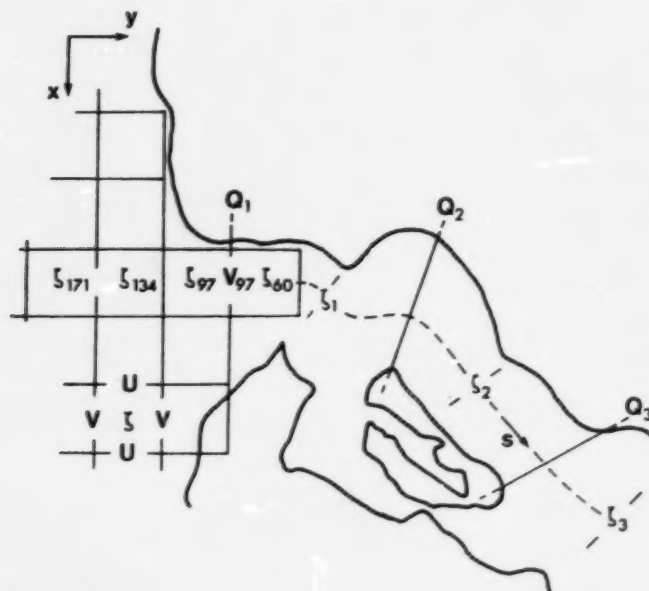


Fig. 8 Junction of the first one-dimensional with the two-dimensional scheme.

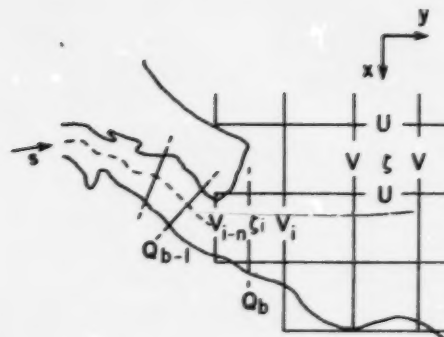


Fig. 9 Junction of the second one-dimensional with the two-dimensional scheme.

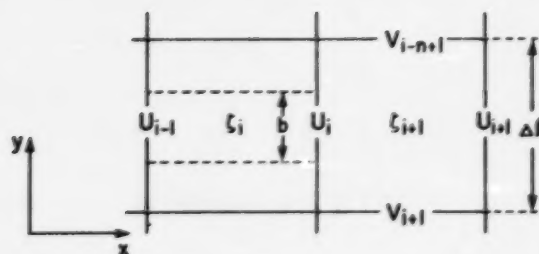


Fig. 10 Grid notation at a narrow pass.

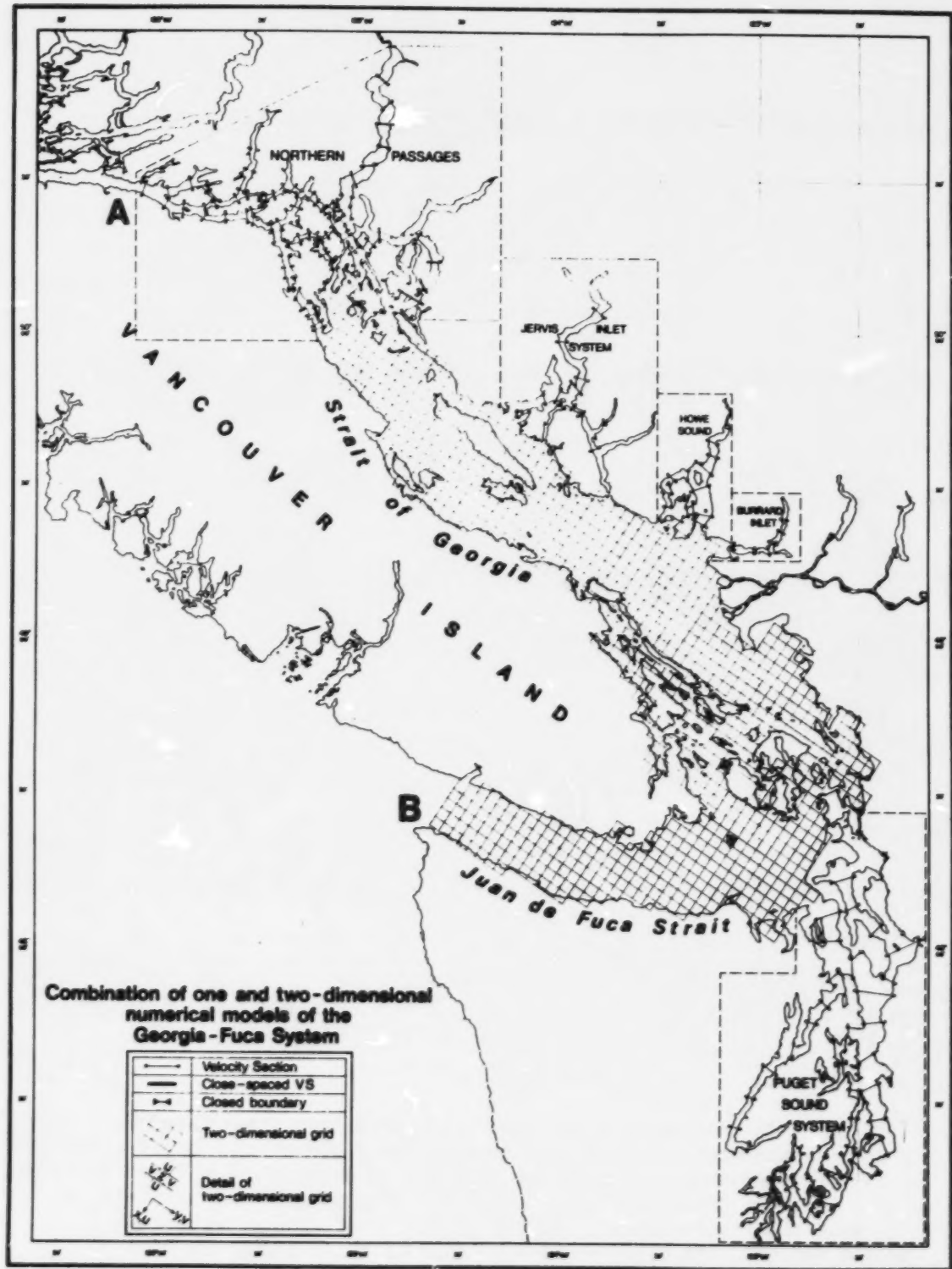


Fig. 11 The overall model comprising joined one- and two-dimensional schemes. Elevations are prescribed at boundary openings A and B.

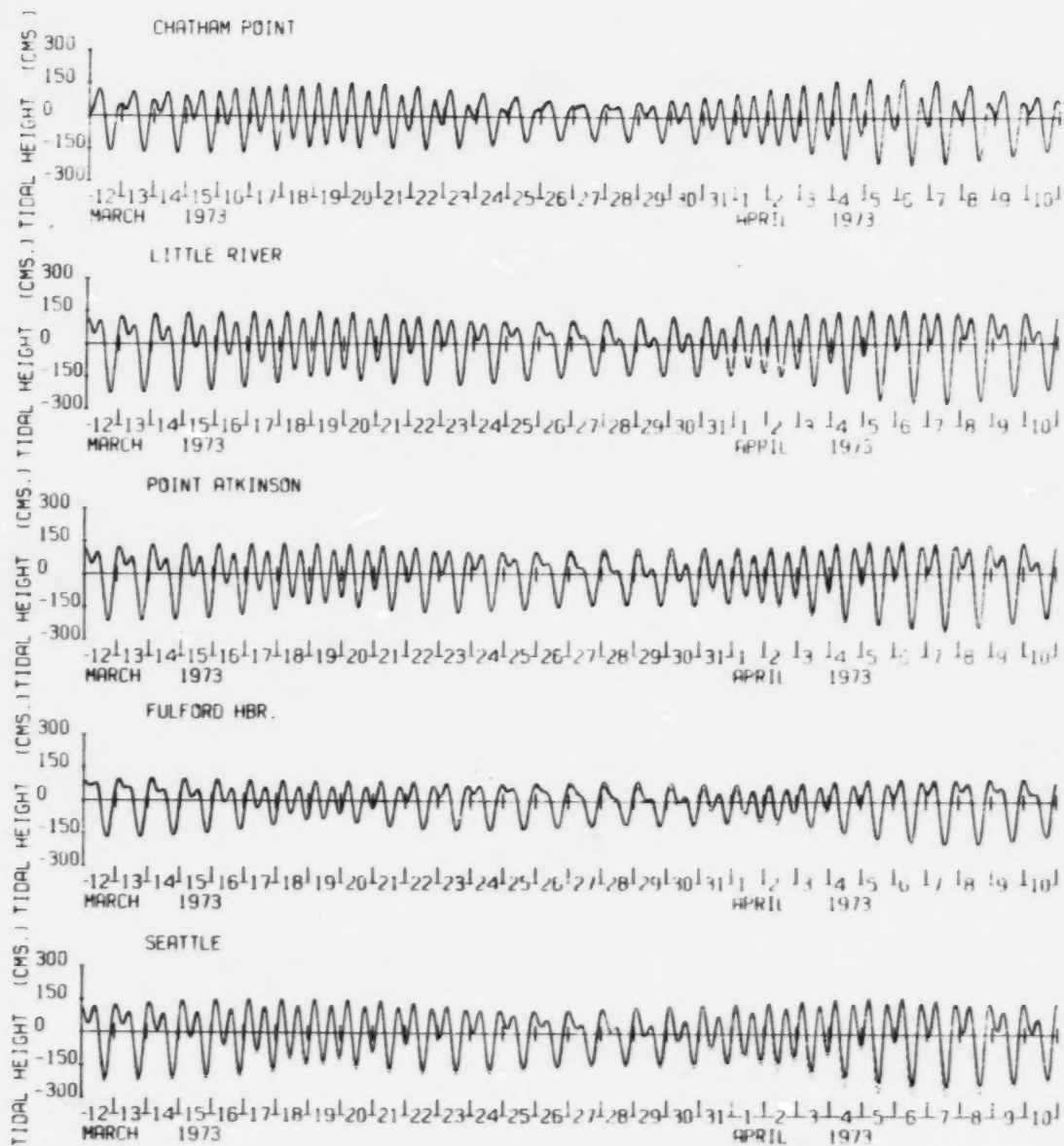


Fig. 12 The computed and observed (i.e. predicted using harmonic constants derived from observations) elevations for five typical locations over a 30-day sequence of mixed tides. These locations are indicated by the underlined tide gauge numbers in Fig. 4.

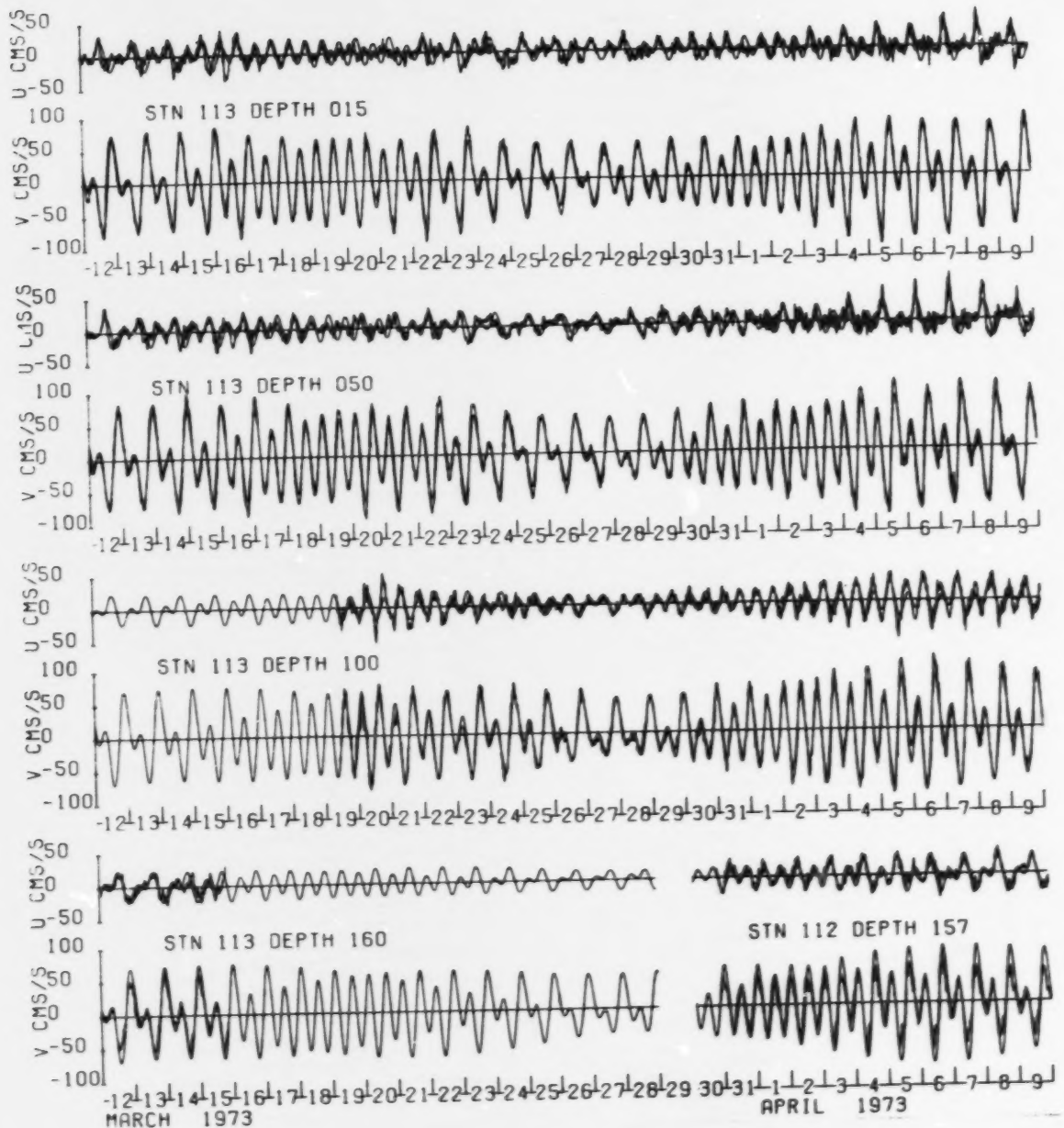


Fig. 13 Comparison of the vertically averaged velocity components (smooth curves) obtained from the model for Station 113 on line 11 (Fig. 4) with observed tidal currents (irregular curves) at four different depths for a 30-day period. These comparisons have been augmented by using further data obtained near the sea bottom at Station 112 (Huggett et al., 1976a).

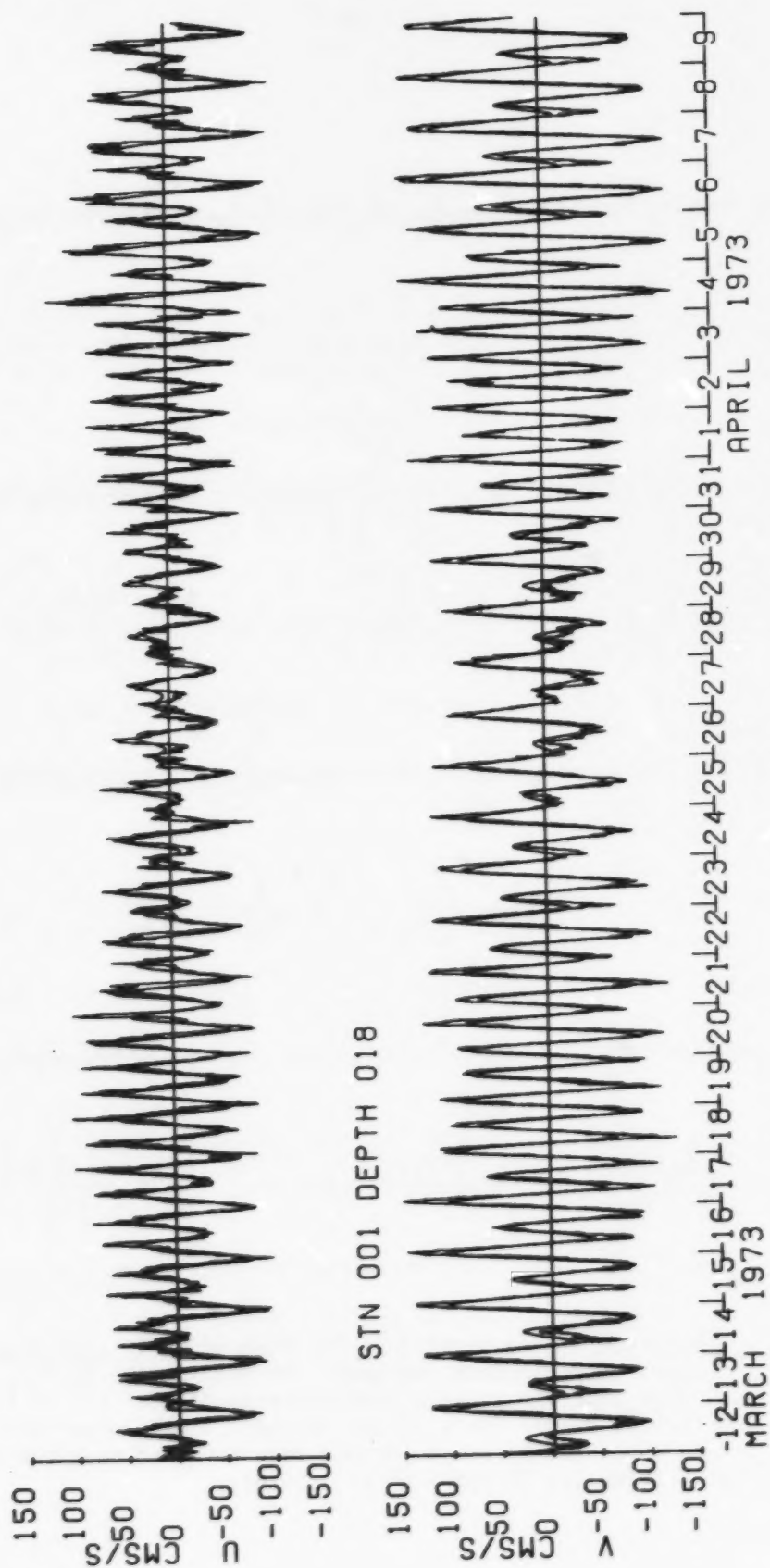


Fig. 14 Comparisons of the computed (smooth curves) and observed (irregular curves) components of tidal velocity at Station 001 (Fig. 4).

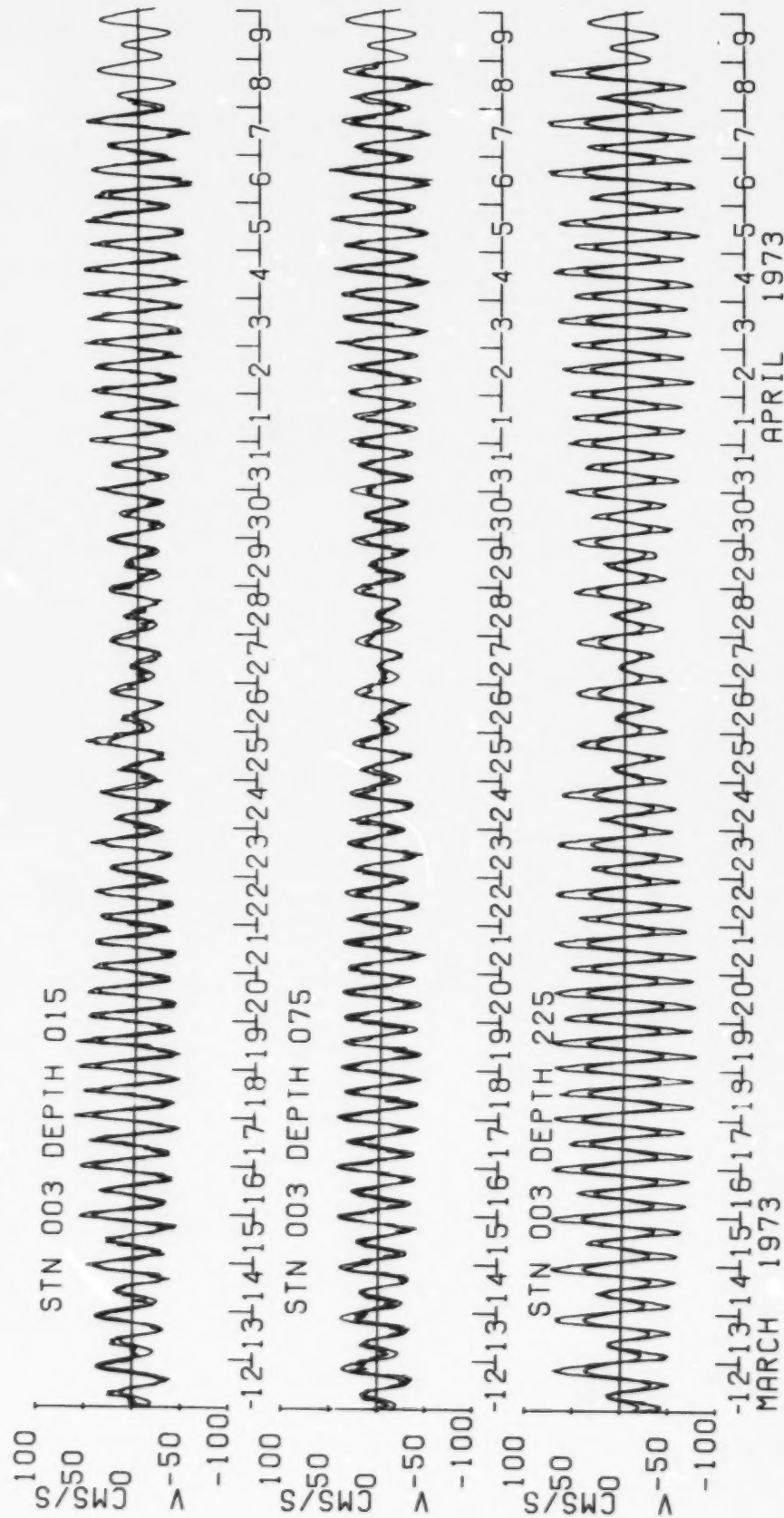


Fig. 15 Comparison of the average velocity over a cross-section (smooth curve) obtained from the model with the observed tidal velocities (irregular curves) at three different depths at a central location in Johnstone Strait, Line 1, Fig. 4, for a 30-day period (Huggett et al., 1976h).

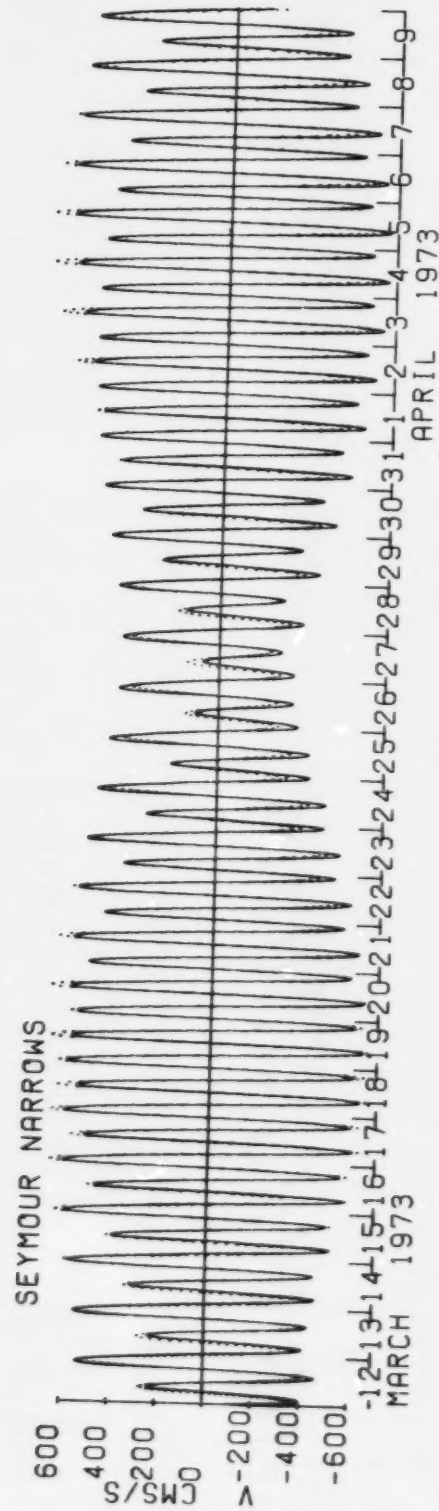


Fig. 16 Comparison of the average velocity (solid line) over a cross-section of Seymour Narrows, Fig. 1, obtained from the model with the corresponding predicted tidal velocities (dotted line) from the Canadian Hydrographic Service for a 30-day period.

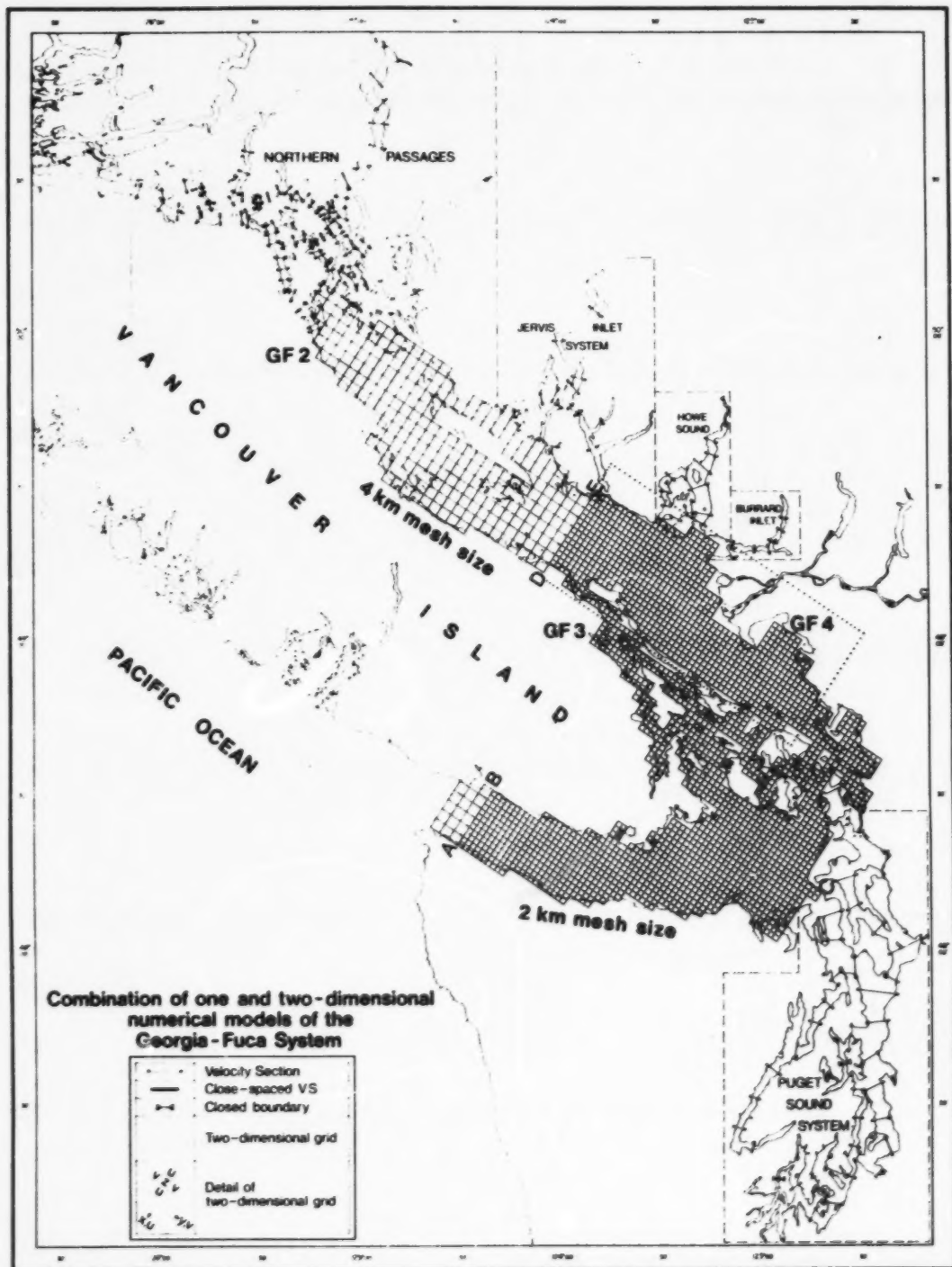


Fig. 17 The fine grid model (GF3) and area covered by the upper layer model (GF4).

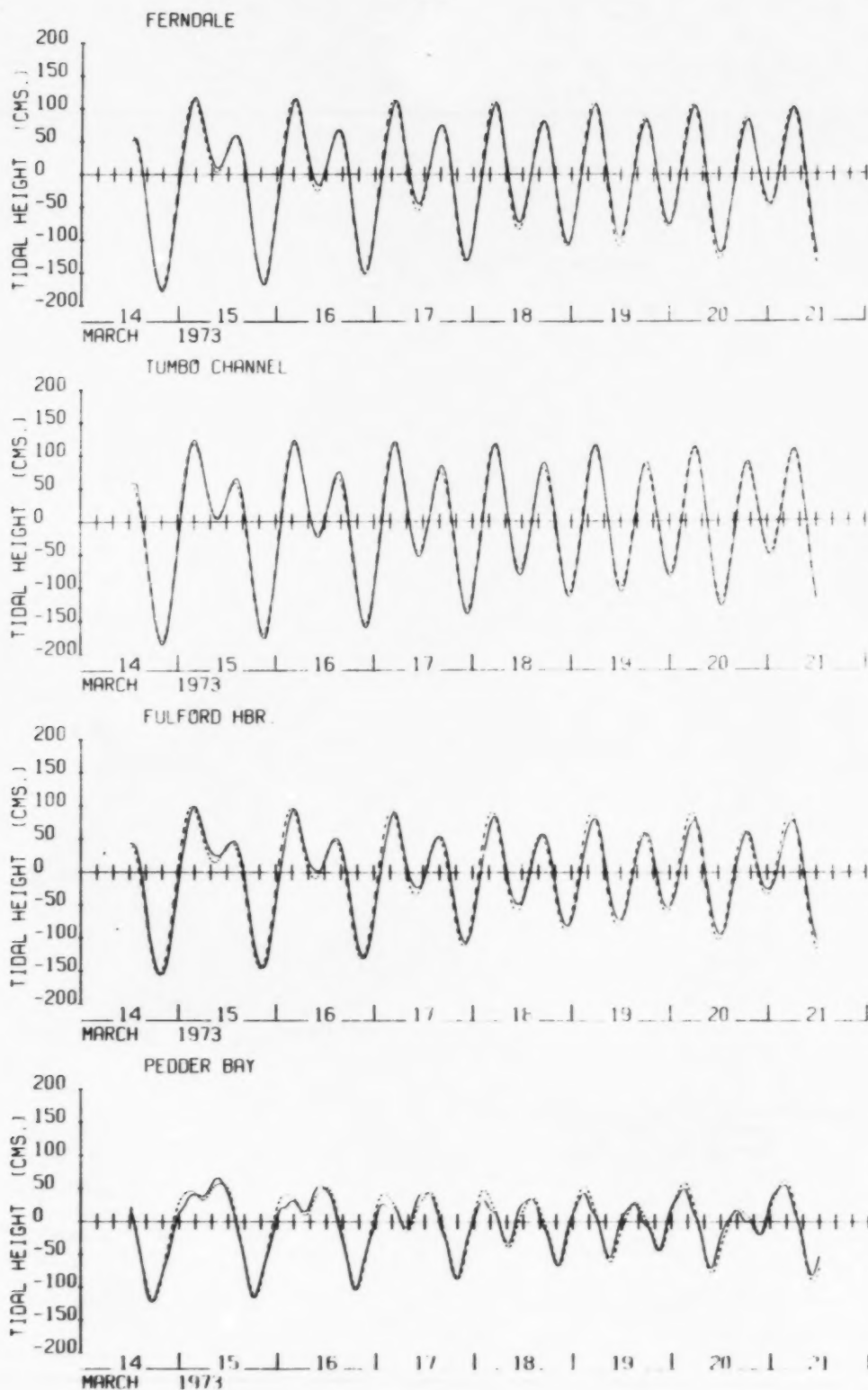


Fig. 18 Observed and computed (GF3) tidal elevations for representative locations.

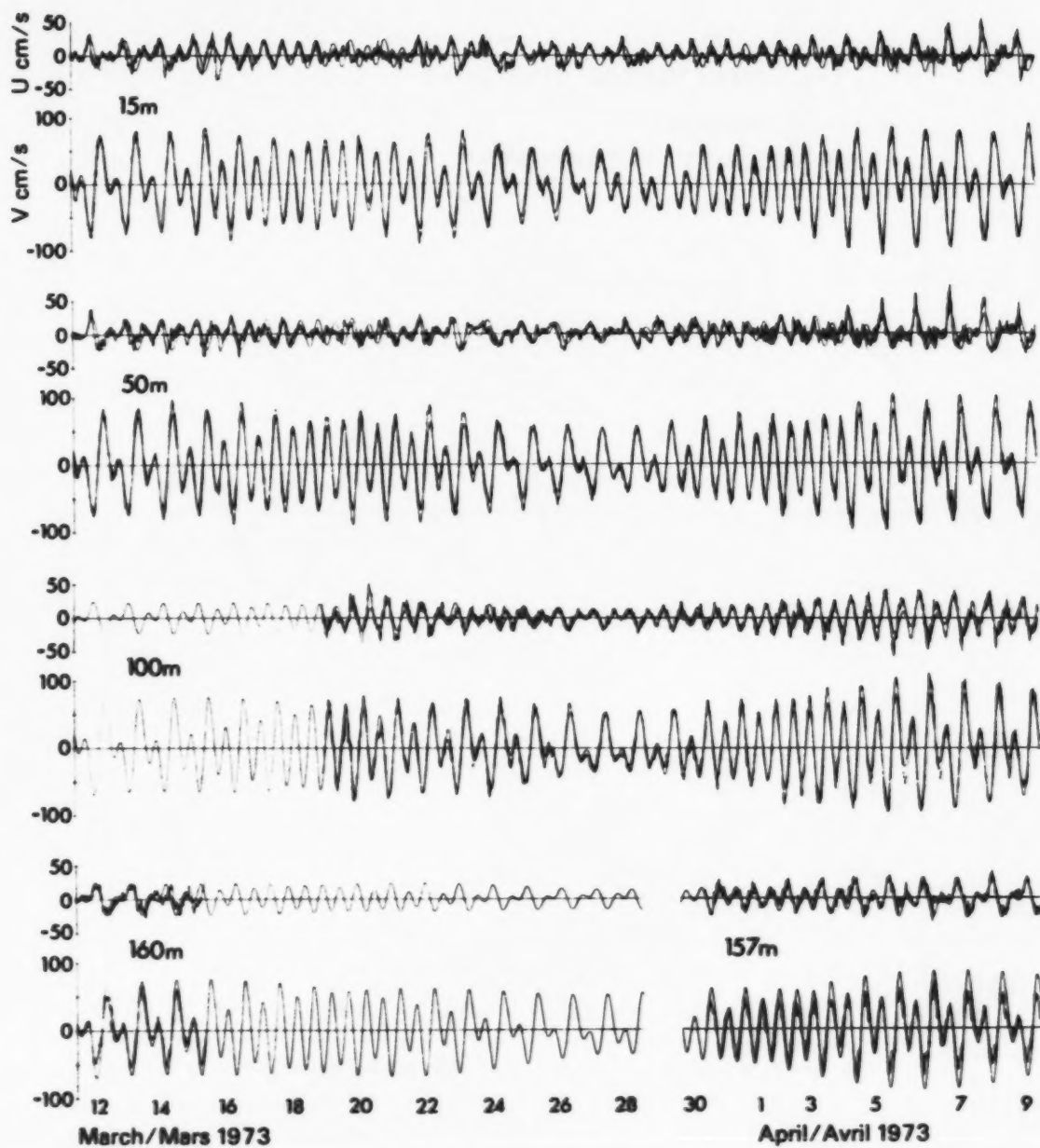


Fig. 19 Representative comparisons of observed and computed (GF3) velocities from line 11 in Juan de Fuca Strait (u cross-channel, v longitudinal, component).

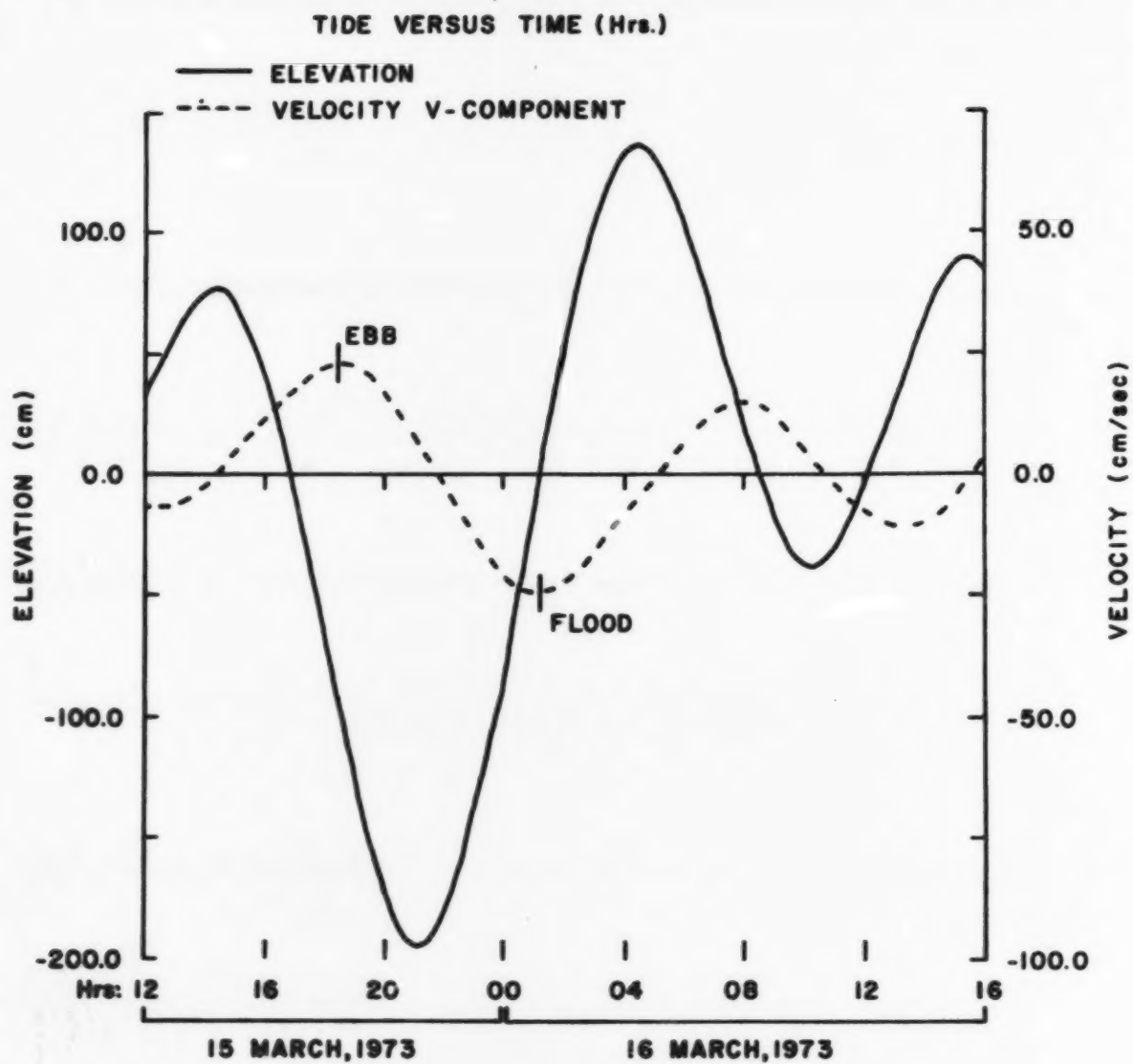


Fig. 20 Tidal elevation and the (dominant) longitudinal component of the tidal stream at a central location in the Strait of Georgia off the mouth of the Fraser River.

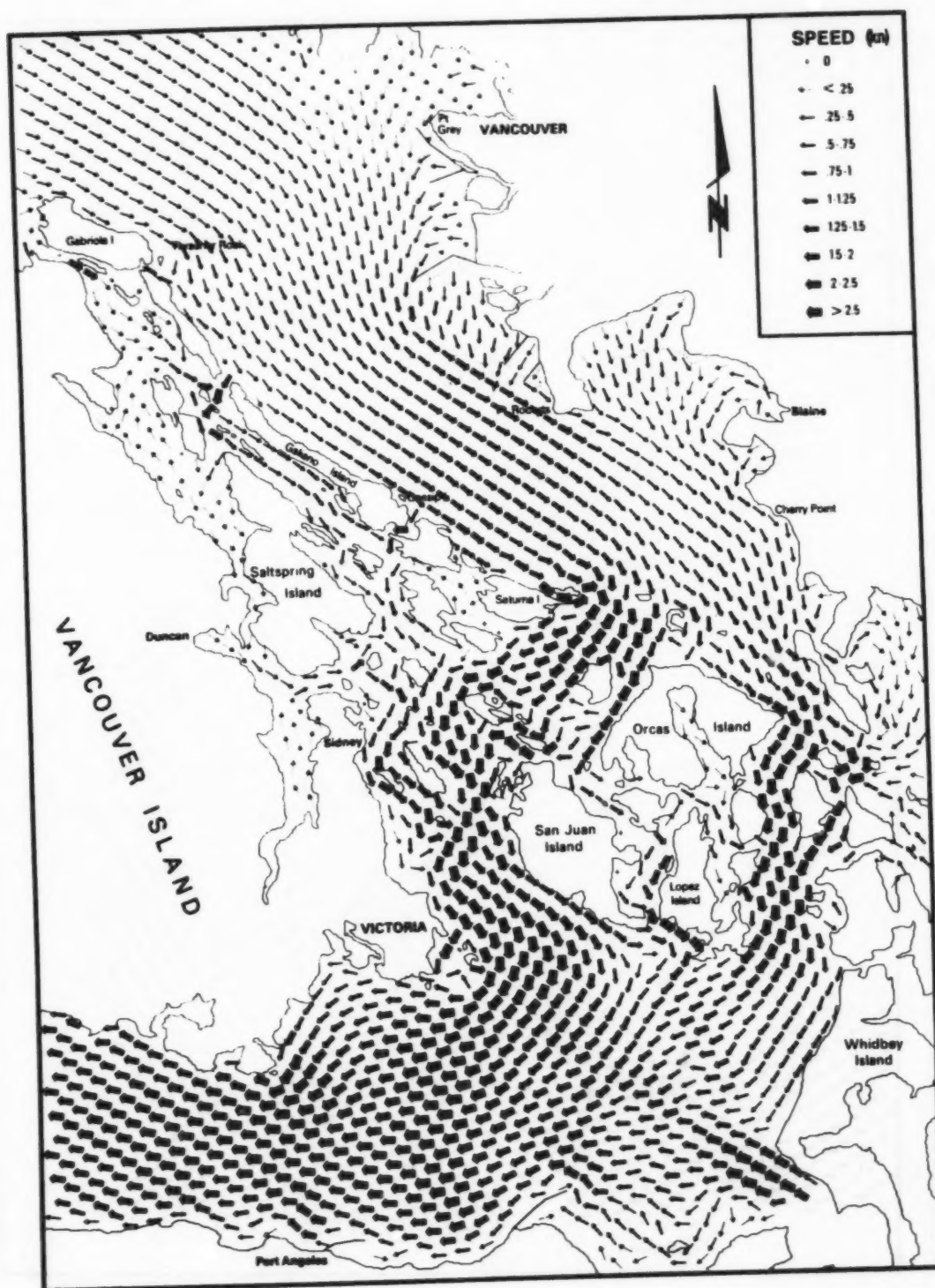


Fig. 21 Ebb velocity vectors obtained from the model (GF3).

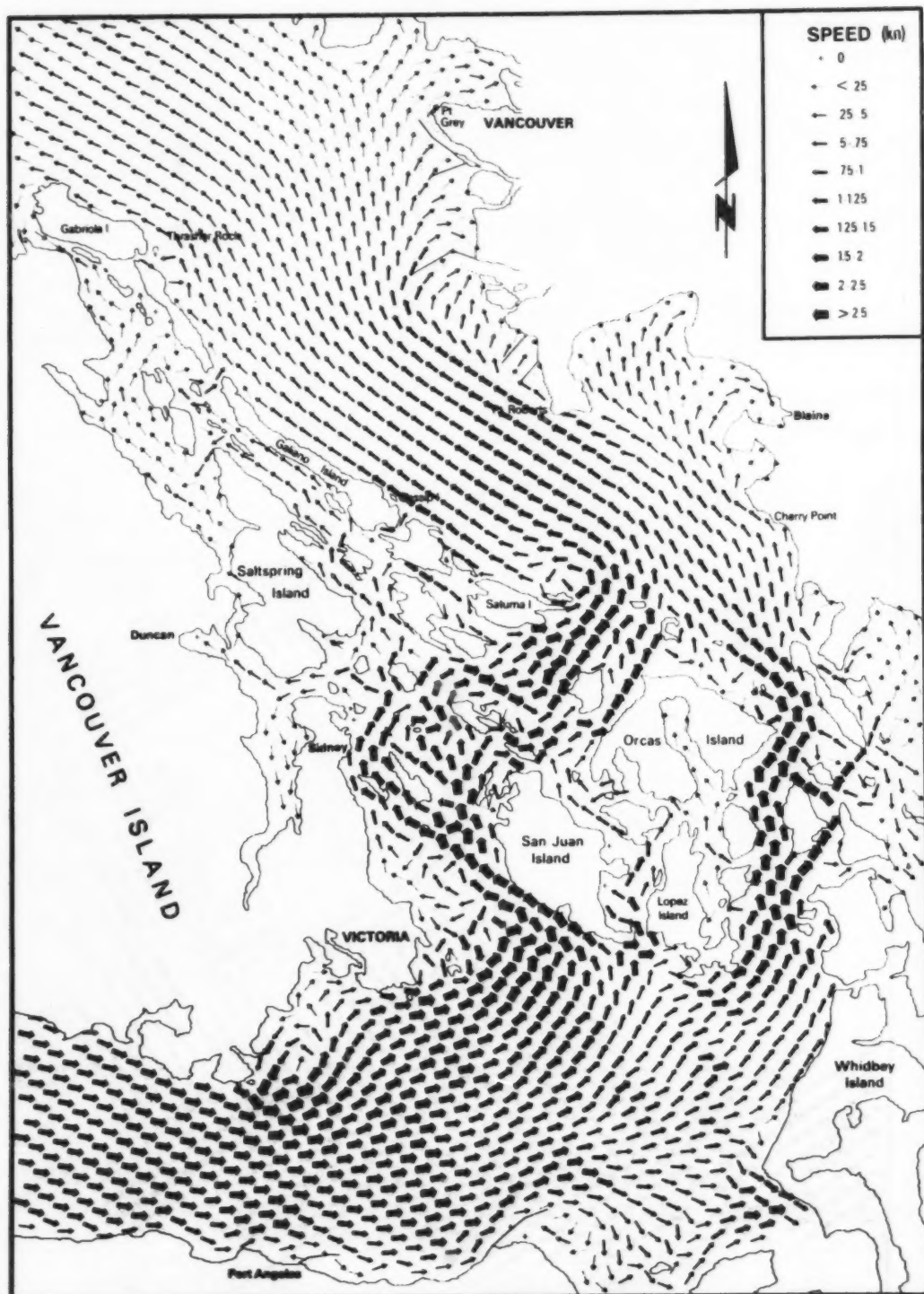


Fig. 22 Flood velocity vectors obtained from the model (GF3).

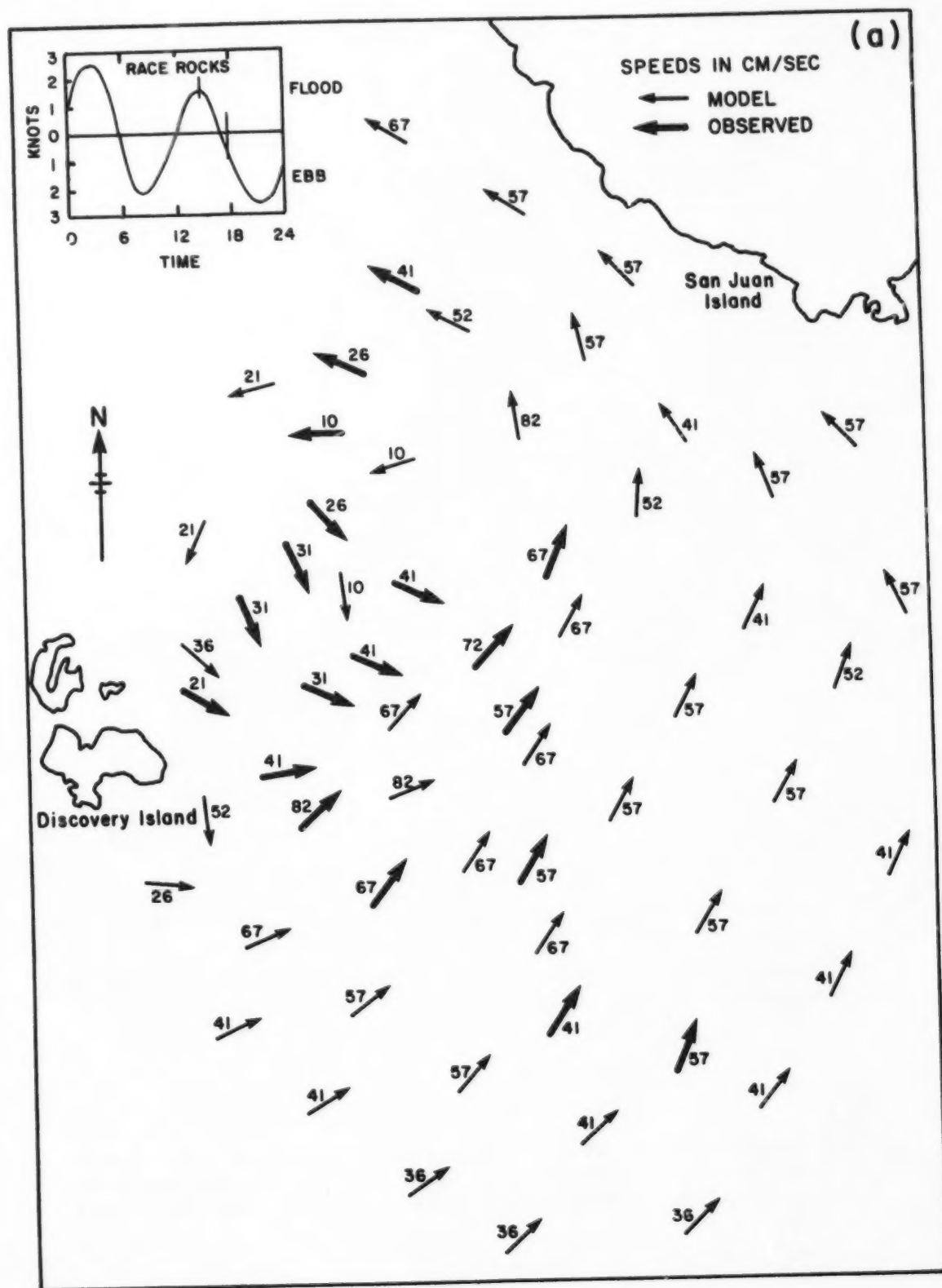


Fig. 23 Unit vectors showing the direction and speeds of the flood tidal stream at the southern end of Haro Strait as obtained from the model (GF3) and from observed drogue tracks.

RESIDUAL CURRENTS

— = 10 CM/SEC

UNIT LENGTH = 3 CM/SEC

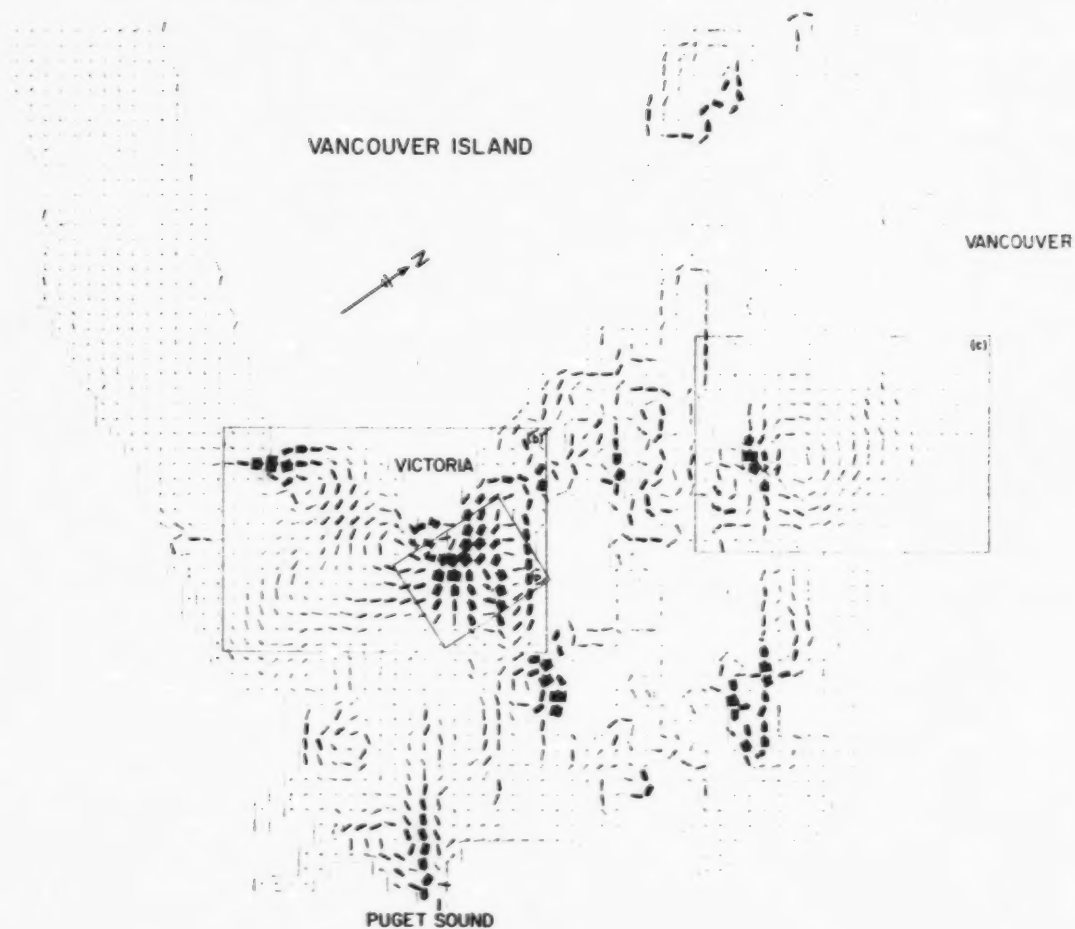


Fig. 24 Distribution of residual tidal velocity vectors obtained from a one month simulation of mixed tides (61 constituents) using the model (GF3). (Insets denote areas included in Figs. 23, 25 and 26).

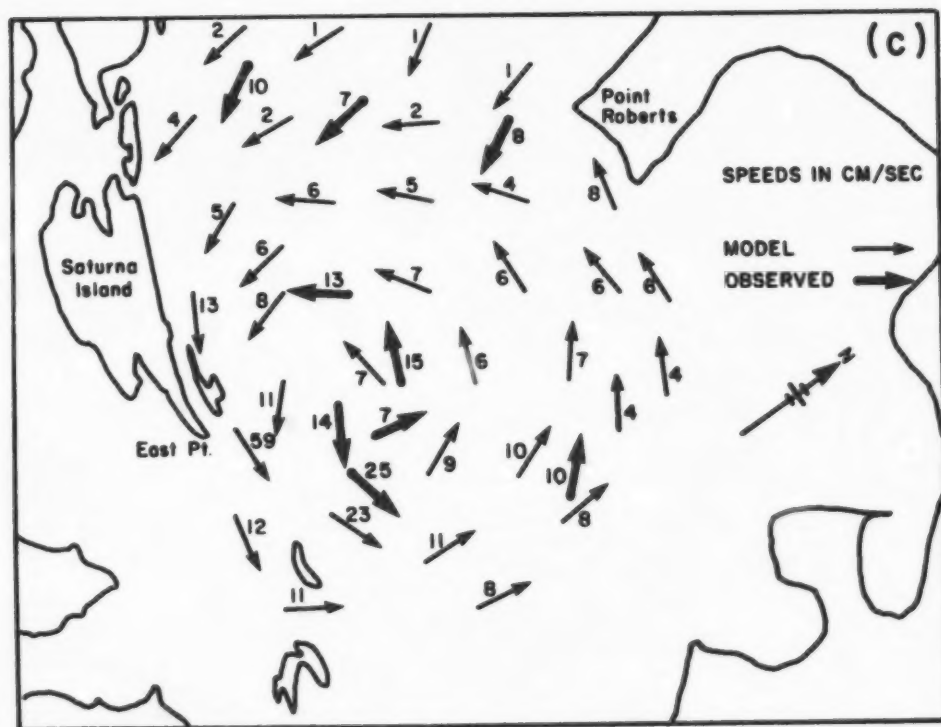


Fig. 25 Comparisons of residual tidal velocities obtained from moored current meter records in the southern part of the Strait of Georgia with those obtained from the model (GF3).

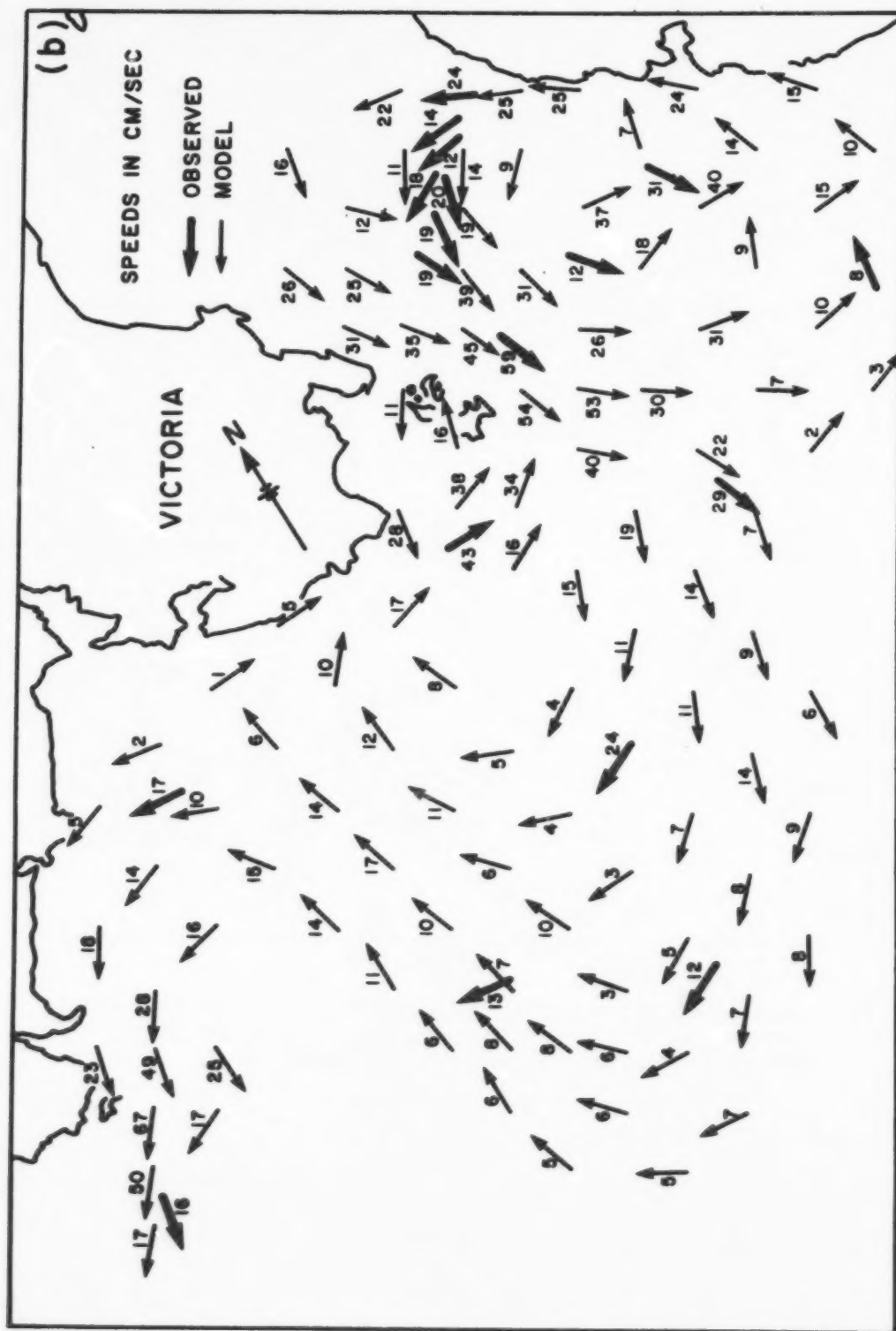


Fig. 26 Comparisons of residual tidal velocities obtained from moored current meters in a region extending from Haro Strait to Race Rocks with those obtained from the model.

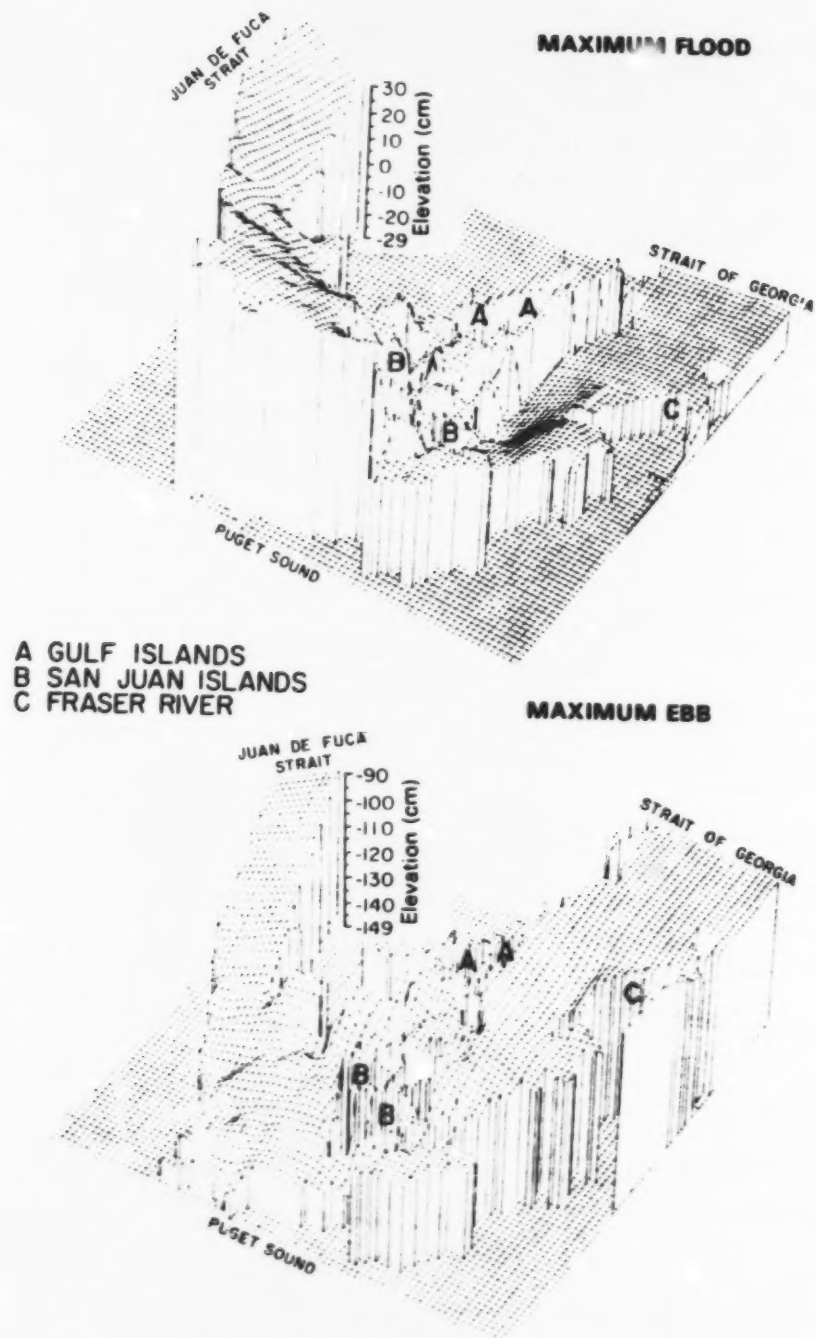


Fig. 27 Three-dimensional diagrams showing the shape of the water surface at the same instant, respectively, as the preceding plots of ebb (Fig. 21) and flood (Fig. 22) vectors.

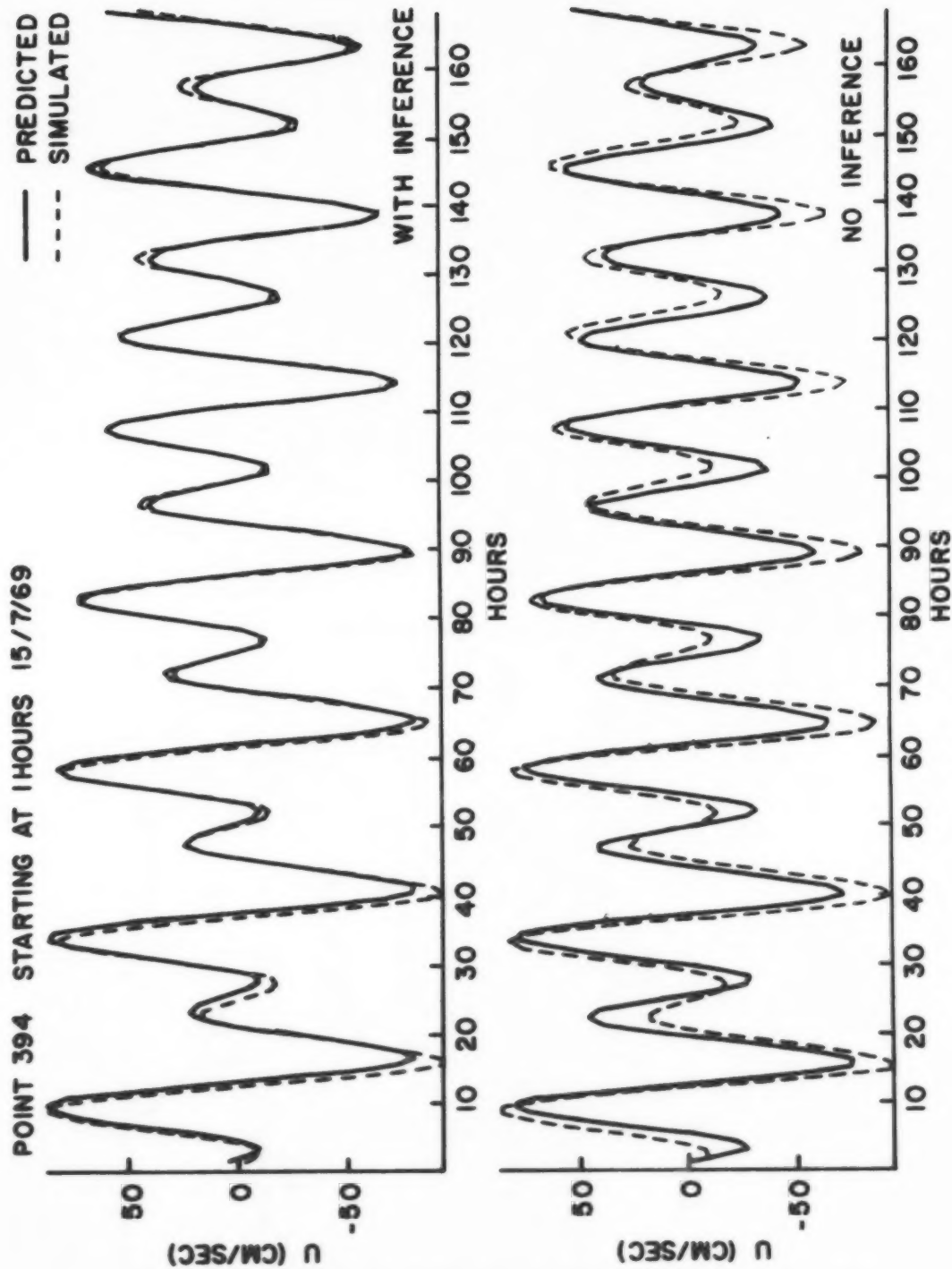


Fig. 28 Typical comparisons of velocity time series, obtained respectively through direct simulation in the model, and through prediction based on harmonic constants from an earlier model run. The plots illustrate the improvement resulting from use of inference in the harmonic analysis. The velocity point is located in the central part of inner Juan de Fuca Strait.

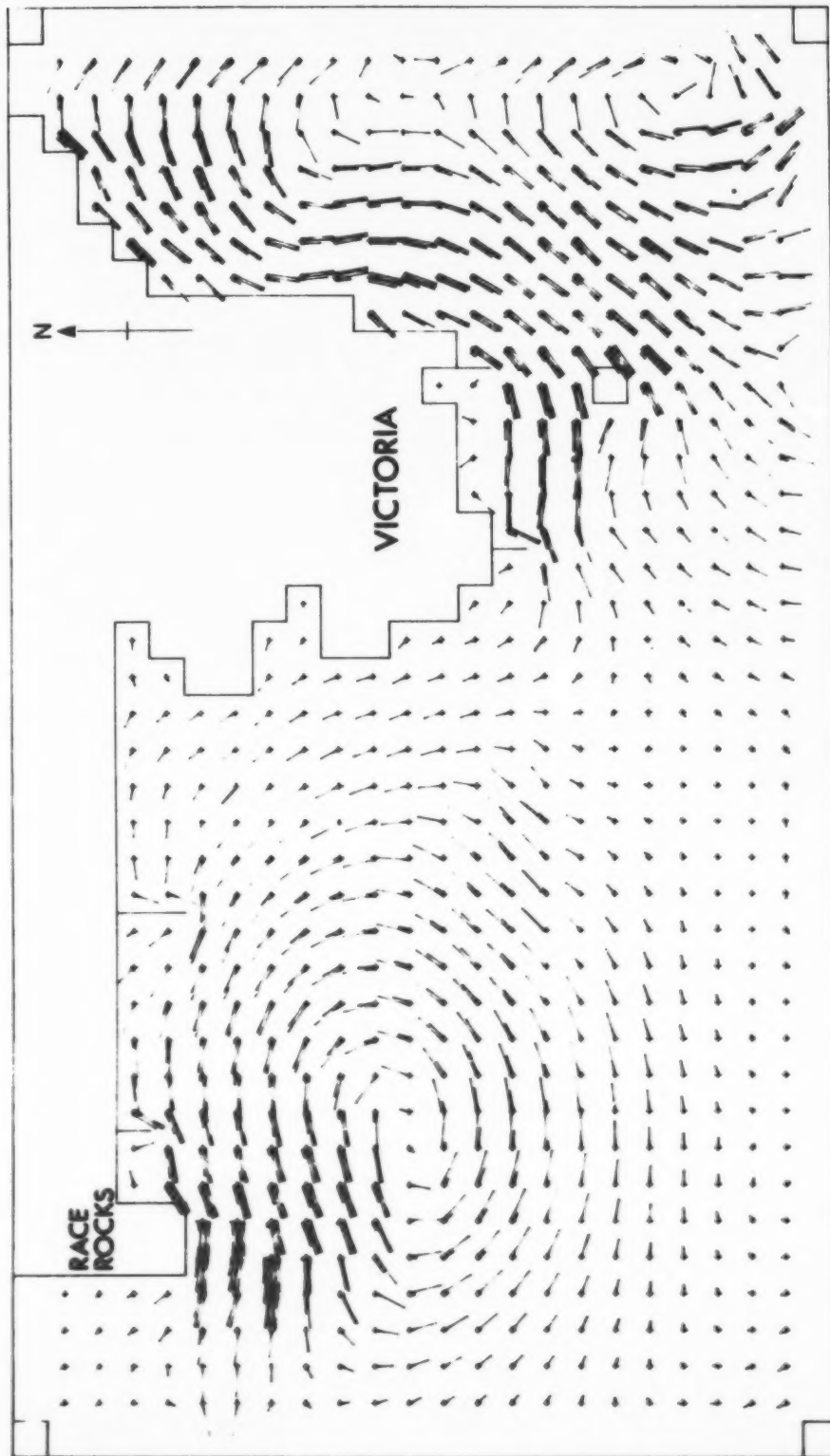


Fig. 29 Distribution of tidal velocity vectors at the beginning of an ebb, obtained from a small area finer resolution (1 km mesh) two-dimensional model, for the region between Race Rocks and Haro Strait, which takes its boundary conditions from the larger overall model GF3.

POOR

QUALITY

ORIGINAL

